

# Fuzzy Multi-Objective Supplier Selection Problem for Multiple Items in a Supply Chain

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**Abstract**-Today's market is highly competitive. The role of supply chain managers is to select best suppliers because major part of the capital is spent on purchasing raw material/semi finished items. The strategic decision of supply chain is to minimize the expenses on the purchase of items. There are several criteria involved in this problem; such as cost, quality, on-time delivery and long term relationship. Some of the criteria are quantitative in nature and some the criteria are qualitative in nature. Qualitative criteria are expressed in triangular fuzzy numbers. It requires defuzzification; the graded mean integration (GMI) representation method is used. Again all of these criteria are conflicting in nature that's why fuzzy programming is used. For formulating the crisp model, it requires defuzzification; fuzzy compensatory operator is applied. This model gives us the idea about supplier selection as well as order quantity from each selected suppliers. Also, the numerical example is given to illustrate the above methods.

**Index Terms**- Fuzzy; Supplier; Supply chain; Multi-Objective; Multi-item

## I. INTRODUCTION

The environment in global market is uncertain. While selecting the best suppliers, this is a splendor challenge in front of management team. Best suppliers mean those suppliers who supply the raw material / semi-finished items with lower cost and higher quality. Items should deliver immediately as per requirement of manufacturing firm [Briggs, 1994].

A long term relationship criterion is very important. Suppose manufacturing firm selects the suppliers by considering above three criteria other than long term relationship. Suppliers send material to manufacturing firm. After sometime, these suppliers stop to send material to the firm. In this situation, there is the problem to the manufacturing firm such as production line be interrupted. Ultimately there is direct financial loss. Because of this reason long term relationship criteria is included in this model. Sometimes manufacturer selects suppliers only on job work basis. All the machineries and dies of manufacturing firm are set up in suppliers firm. In this kind of deal, if suppliers stop to send material to the manufacturer then there is also a loss to the manufacturer. Loss due to the cost incurred to set up dies and machineries in the suppliers plant. That's why long term relationship is the best criteria in this model.

Zimmerman (1978;1987) first used the Bellman and Zadeh (1970) method to solve fuzzy goals and fuzzy constraints are treated equivalently Tiwari et al. (1986). Amid et al. (2005) developed multi-objective supplier selection model. Also, Kumar et al. (2004; 2005) and

Dulmin et al. (2003) studied the vendor selection model. Relationship between suppliers and manufacturing plants is very important for effective management of supply chain [Lee et al. 1992].

Leberling (1980) discussed how to obtain the compromise solution. i.e. optimum solution of one objective function is not a optimum solution of the other objective function. Because of this, there is need of efficient solution. Dickson (1996) and Briggs (1994) studied the vendor selection and analysis. Supply chain is the integrated activity, procurement of raw material, convert these raw materials into finished products and then distribute to the warehouses and retailers [Selim et al. 2006 and Narasimhan 1983]. Some academician studied on multiple attribute decision making criteria. Chen et al. (2005) developed supplier selection model. Success of supply chain management depends on its suppliers [Choi et al. 1996].

Supplier selection is a multi-criterion decision making problem under uncertain environments. Therefore, it is reasonable to use fuzzy set theory and Dempster and Shafer theory of evidence (DST). Here, the main idea of the technique for order preference by similarity to an ideal solution (TOPSIS) is developed to deal with supplier selection problem. The basic probability assignment (BPA) can be evaluated by the distance to positive ideal solution and negative ideal solution. Dempster combination rule is used to concatenate the entire criterion. The performance of criterion can be represented as crisp number or fuzzy number according to the real situation (Deng et al. 2011).

This model can be used as a decision support system by the DM to decide what order quantity to place with each supplier in the case of multiple sourcing of multiple item. In a real situation, for a supplier selection problem, most of the information is not known precisely (Amid et al. 2009). It is very difficult to take decision. Such vague terms are "very good in quality", "very poor in on time delivery" and "medium in long term relationship". Deterministic models cannot take this vagueness into account. The ratings of qualitative criteria are expressed as linguistic variables. The linguistic variables are expressed in triangular fuzzy numbers as in Table 2.1.

In order to develop the fuzzy multi-objective supplier selection model, the cost criteria is expressed in quantitative in nature but quality, on time delivery and long term relationship is expressed in qualitative in nature; these criteria are converted into the triangular fuzzy numbers. It requires defuzzification to convert it

into quantitative criteria; the graded mean integration representation method is used. All of these criteria are conflicting in nature, Multi-objective LP formulation requires defuzzification. The method of graded mean integration representation is applied. In supplier selection process many criteria are conflicting with each other. Therefore decision-making process becomes complicated. In this project, Werner's "fuzzy and" ( $\mu_{and}$ ) operator is used to get both compensatory and strongly efficient solution (Ozkok et al. 2011). The beauty of this research paper is that it requires fuzzification twice. Firstly linguistic variables are converted into triangular fuzzy numbers and then process of defuzzification is done by using GMI. The paper is organized as follows. Section 2 includes the basic definition and notations of fuzzy number and linguistic variable. Section 3 describes the proposed multi-objective supplier selection model and section 4 describes the fuzzy membership function as well as model algorithm. The proposed method is illustrated with an example. At the end conclusions are given.

## II. PRELIMINARIES

Following is some definitions and notations that are used throughout the paper (Kauffman et al. 1985; Zimmermann, 1991).

**Definition 2.1.** In fuzzy sets, each elements is mapped to  $[0,1]$  by membership function.

$$\mu_{\tilde{A}}(x): X \rightarrow [0, 1]$$

Where  $[0, 1]$  means real numbers between 0 and 1.

**Definition 2.2.** A fuzzy set  $\tilde{A}$  of the universe of discourse  $X$  is convex if and only if  $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$   $\forall x_1, x_2 \in X, \lambda \in [0, 1]$ . Where min denotes minimum operator (Klir et al. 1995).

**Definition 2.3.** A fuzzy  $\tilde{A}$  of the universe of discourse  $X$  is called a normal fuzzy set implying that

$$\exists x_i \in X, \mu_{\tilde{A}}(x_i) = 1.$$

**Definition 2.4.** If a fuzzy set is convex and normalized, and its membership function is defined in  $\mathfrak{R}$  and piecewise continuous, it is called as fuzzy number.

**Definition 2.5.** Membership value of member  $x$  in the union takes the greater value of membership between  $\tilde{A}$  and  $\tilde{B}$

$$\mu_{\tilde{A} \cup \tilde{B}}(x) = \max[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] \quad \forall x \in X.$$

**Definition 2.6.** Intersection of fuzzy sets  $\tilde{A}$  and  $\tilde{B}$  takes smaller value of membership function between  $\tilde{A}$  and  $\tilde{B}$

$$\mu_{\tilde{A} \cap \tilde{B}}(x) = \min[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] \quad \forall x \in X.$$

**Definition 2.7.** A triangular fuzzy number  $\tilde{n}$  can be represented by triplet  $(n_1, n_2, n_3)$  shown in Fig. 1. The membership function  $\mu_{\tilde{n}}(x)$  is defined as:

$$\mu_{\tilde{n}}(x) = \begin{cases} 0, & x < n_1 \\ \frac{x - n_1}{n_2 - n_1}, & n_1 \leq x \leq n_2 \\ \frac{n_3 - x}{n_3 - n_2}, & n_2 \leq x \leq n_3 \\ 0, & x > n_3 \end{cases}$$

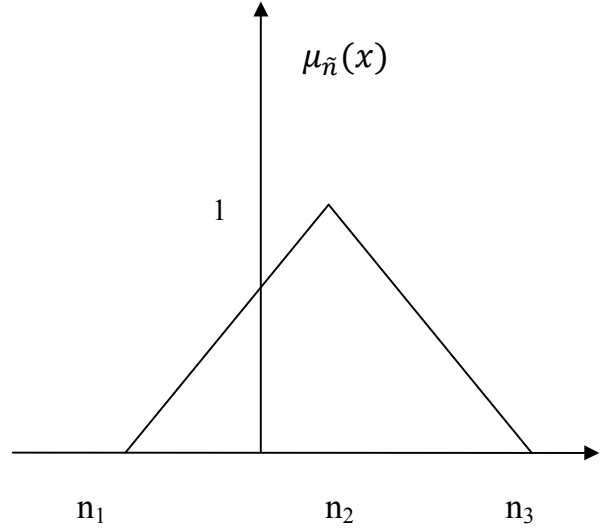


Fig. 1 Triangular fuzzy number

## 2.8. Linguistic Variable:

The concept of linguistic variable is very useful in dealing with situations which are too complex or ill-defined. Linguistic variables are expressed in words or sentences or artificial languages, where each linguistic value can be modelled by a fuzzy set (Kauffman et al. 1985). In this paper, the ratings of qualitative criteria are expressed as linguistic variables. The linguistic variables are expressed in triangular fuzzy numbers as in Table 1.

Table 1: Linguistic Variables for criteria

Linguistic Variables	Weight
Very poor (VP)	(0, 0.2, 0.4)
Poor (P)	(0.3, 0.5, 0.6)
Medium (M)	(0.6, 0.7, 0.8)
Good (G)	(0.7, 0.8, 0.9)
Very good (VG)	(0.8, 0.9, 1.0)

It should be noticed that there are many different methods to represent linguistic items. Which kind of representation method is used? It depends on the real application systems and domain experts' opinions.

## 2.9. Defuzzification:

The defuzzification entails converting the fuzzy value into a crisp value and determining the ordinal positions of  $n$ -fuzzy input parameter vector. There are several defuzzification techniques (Zimmermann 1991), but some of the most widely used techniques, such as centre of area, first of maximums, last of maximums and middle of maximums.

In this paper, the canonical representation of operation on triangular fuzzy numbers [Chou 2003], which is based on the graded mean integration representation method is used in defuzzification.

- Graded mean integration representation of triangular fuzzy number:

Given a triangular fuzzy number  $\tilde{A} = (a_1, a_2, a_3)$ , the graded mean integration representation of triangular fuzzy number  $\tilde{A}$  is defined as

$$p(\tilde{A}) = \frac{1}{6}(a_1 + 4a_2 + a_3)$$

Table 2: Graded mean integration (GMI) representation for the ratings of each criterion

Linguistic Variables	GMI
Very poor (VP)	0.20
Poor (P)	0.48
Medium (M)	0.70
Good (G)	0.80
Very good (VG)	0.90

### III. MULTIOBJECTIVE SUPPLIER SELECTION MODEL

In order to formulate the model, the following notations are defined.

- $n$  number of suppliers.
- $U$  number of items.
- $c_{iu}$  cost of the  $u^{\text{th}}$  purchased item from  $i^{\text{th}}$  suppliers.
- $q_{iu}$  quality of the  $u^{\text{th}}$  purchased item from  $i^{\text{th}}$  suppliers.
- $t_{iu}$  on time delivery of an  $u^{\text{th}}$  item from  $i^{\text{th}}$  suppliers.
- $s_{iu}$  long term relationship of  $i^{\text{th}}$  suppliers for  $u^{\text{th}}$  item to manufacturing firm.
- $D_u$  demand over period for  $u^{\text{th}}$  item.
- $p_{iu}$  capacity of  $i^{\text{th}}$  suppliers to supply the  $u^{\text{th}}$  item.
- $B_i$  total available budget of  $i^{\text{th}}$  suppliers.

A multi-objective model for supplier selection problem can be stated as follows:

Objective function of net cost for ordering the aggregate demand:

$$\begin{aligned} & \text{Minimize } Z_1 \\ & = \sum_{i=1}^n \sum_{u=1}^U c_{iu} x_{iu} \end{aligned}$$

Objective function of quality of items of the suppliers:

$$\begin{aligned} & \text{Maximize } Z_2 \\ & = \sum_{i=1}^n \sum_{u=1}^U q_{iu} x_{iu} \end{aligned}$$

Objective function of the on-time delivery of items of the suppliers

$$\begin{aligned} & \text{Maximize } Z_3 \\ & = \sum_{i=1}^n \sum_{u=1}^U t_{iu} x_{iu} \end{aligned}$$

Objective function of the long-term relationship of the suppliers:

$$\begin{aligned} & \text{Maximize } Z_4 \\ & = \sum_{i=1}^n \sum_{u=1}^U s_{iu} x_{iu} \end{aligned} \tag{1}$$

Subject to constraint

$$\begin{aligned} & \sum_{i=1}^n x_{iu} \\ & = D_u \quad \forall u \end{aligned}$$

Constraint is due to aggregate demand of item.

$$\begin{aligned} & x_{iu} \leq p_{iu} \quad , \quad i \\ & = 1, 2, \dots, n. \quad \forall u. \end{aligned}$$

Constraint is due to the maximum capacity of the suppliers.

$$\begin{aligned} & \sum_{u=1}^U c_{iu} x_{iu} \leq B_i \quad , \quad i \\ & = 1, 2, \dots, n. \end{aligned}$$

Constraint is due to budget allocated to the suppliers.

$$\begin{aligned} & x_{iu} \geq 0 \text{ and integer} \quad , \quad i \\ & = 1, 2, \dots, n. \quad \forall u. \end{aligned}$$

Constraint is nonnegative restriction and all order quantities are integers.

### IV. FUZZY MEMBERSHIP FUNCTION

The linear membership function for fuzzy objectives is given as, we define membership function for the  $m^{\text{th}}$  objective function (Minimization type) as follows:

$$\mu_{Z_m}(x) = \begin{cases} 1 & \text{for } Z_m(x) \leq Z_m^l \\ \frac{Z_m^{up} - Z_m(x)}{Z_m^{up} - Z_m^l} & \text{for } Z_m^l \leq Z_m(x) \leq Z_m^{up} \\ 0 & \text{for } Z_m(x) \geq Z_m^{up} \end{cases} \tag{2}$$

Here  $Z_m^{up}$  is  $\max_m Z_m(x^*)$  and  $Z_m^l$  is  $\min_m Z_m(x^*)$ , hence  $x^*$  is optimum solution.

Membership function for the  $k^{\text{th}}$  objective function (Maximize) is as follows:

$$\mu_{Z_k}(x) = \begin{cases} 1 & \text{for } Z_k(x) \geq Z_k^{up} \\ \frac{Z_k(x) - Z_k^l}{Z_k^{up} - Z_k^l} & \text{for } Z_k^l \leq Z_k(x) \leq Z_k^{up} \\ 0 & \text{for } Z_k(x) \leq Z_k^l \end{cases} \tag{3}$$

Here  $Z_m^{up}$  is  $\max_k Z_k(x^*)$  and  $Z_k^l$  is  $\min_k Z_k(x^*)$ , Hence  $x^*$  is optimum solution.

These membership functions are illustrated in Fig. 2.1 and Fig. 2.2 respectively.  $Z_m^0$  and  $Z_k^0$  are the aspiration level that the decision maker wants to reach

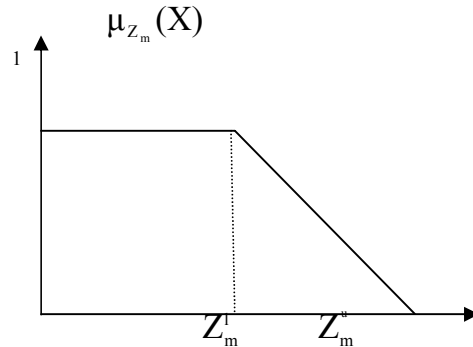


Fig. 2.1

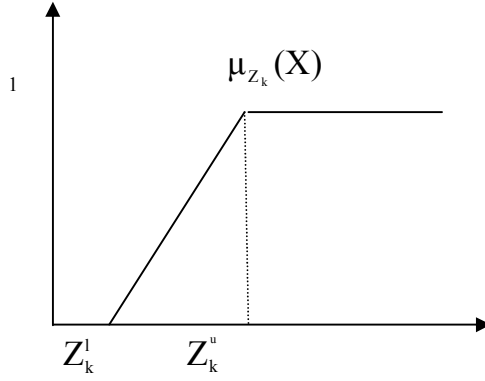


Fig. 2.2

Fig.2. Membership function for fuzzy objective functions

We find an equivalent crisp model by using a linear membership function for the initial fuzzy model using equation (1, 2 and 3).

Maximize  $\lambda$

Subject to constraints

$$\begin{aligned} Z_m(x) + \lambda(Z_m^{up} - Z_m^l) &\leq Z_m^{up}, & m \\ &= K + 1, K + 2, \dots, M. \\ Z_k(x) - \lambda(Z_k^{up} - Z_k^l) &\geq Z_k^l, & k \\ &= 1, 2, \dots, K. \end{aligned} \quad (4)$$

$$\begin{aligned} g(x) &\leq b, \\ x &\geq 0 \text{ and integer.} \end{aligned}$$

In this paper, Werner's compensatory "fuzzy and" operator is used and show that the solutions generated by this operator do guarantee pareto-optimality for this problem. Werner's (1998) introduced the compensatory "fuzzy and" operator which is the convex combination of minimum and arithmetical mean:

$$\begin{aligned} \mu_{and} \\ = \lambda \\ + \frac{(1-\gamma)}{J} \sum_{j=1}^J \lambda_j \end{aligned}$$

where  $0 \leq \lambda \leq 1$ ,  $j = 1, 2, \dots, J$  and magnitude of  $\gamma \in [0, 1]$  represent grade of compensation.

It pointed out that Zimmermann's min operator model doesn't always yield a strongly-efficient solution (Guu et al. 1997; Guu et al. 2001). By using Werner's  $\mu_{and}$  operator, (4) is converted to as follows:

$$\text{Maximize } \mu_{and} = \lambda + \frac{(1-\gamma)}{J} \sum_{j=1}^J \lambda_j$$

Subject to constraints

$$\begin{aligned} Z_m(x) + (\lambda + \lambda_j)(Z_m^{up} - Z_m^l) &\leq Z_m^{up}, & m \\ &= K + 1, K + 2, \dots, M. \\ Z_k(x) - (\lambda + \lambda_j)(Z_k^{up} - Z_k^l) &\geq Z_k^l, & k \\ &= 1, 2, \dots, K. \\ g_r(x) &\leq b_r, & r \\ &= 1, 2, \dots, R. \end{aligned} \quad (6)$$

$$\begin{aligned} \lambda + \lambda_j &\leq 1 \\ \lambda, \quad \forall \lambda_j &\in [0, 1], \quad j = 1, 2, \dots, J \\ \gamma &\in [0, 1] \quad x \geq 0 \text{ and integer.} \end{aligned}$$

### A. Model Algorithm

Step I: Set the committee of decision makers. Some of the criteria are qualitative nature and some of criteria are quantitative in nature.

Step II: Assign the interval weights to criteria as well as decision makers (DM).

Step III: Obtain the graded mean integration (GMI) of all the qualitative criteria by using the following formula.

$$P(\tilde{A}) = \frac{1}{6}(a_1 + 4a_2 + a_3)$$

Step IV: Obtain the individual weights of criteria and DM's by using the formula.

$$W_j = \frac{L_j + U_j}{\sum_{j=1}^n L_j + U_j}$$

Where  $L_j$  and  $U_j$  are lower and upper limit of interval weights respectively.

Step V: Combine the weights; multiply DM's weight by the criteria weight.

Step VI: Obtain the weighted average of all the criteria for each supplier.

Step VII: Formulate the multi-objective linear programming problem.

Step VIII: Solve the multi-objective linear programming problem for each objective. We get the lower and upper bound. Also obtain the difference of bounds.

Step IX: Again formulate the crisp LPP and maximize the Werner's  $\mu_{and}$  operator subject to all the constraints as well as all the objectives.

Step X: Solve this problem by using usual simplex algorithm for different values of compensatory operator ( $\gamma$ ) (Use LINGO software). We get the best efficient solution.

### B. Numerical Example

Lock manufacturing company desires to select raw material suppliers for Lock type A & B. After screening, five suppliers ( $S_1, S_2, S_3, S_4, S_5$ ) left for evaluation. A group of decision-makers,  $D_1, D_2$  and  $D_3$ , has been formed to select most suitable supplier. Four criteria are considered:

- (1) Cost of a supplied item ( $C_1$ ).
- (2) Quality of a supplied item ( $C_2$ ).
- (5) (3) On time delivery of a supplied item ( $C_3$ ).
- (4) Long term relationship with supplier ( $C_4$ ).

The hierarchical structure of this decision problem is shown in Fig.3.

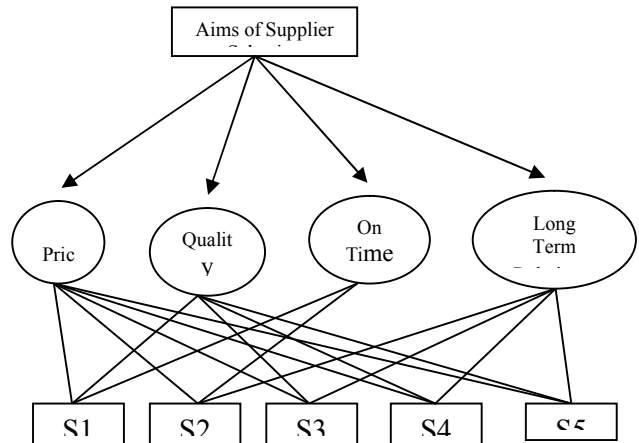


Fig. 3. Hierarchical Structure of Supplier Selection  
 After, examining the suppliers following information is collected. Demand over period are 2000 for Lock type A and 3000 for Lock type B.

Table 3:

Supplier s	S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>	S <sub>5</sub>
Capacity for Lock Type A	800	700	750	850	650
Capacity for Lock Type B	1200	1000	500	700	900
Budget	2,00,000	3,00,000	5,00,000	4,00,000	2,00,000

Table 4: Data for national level supplier selection for lock type A:

	Performance	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>
D <sub>1</sub> [0.10, 0.25]	Weight	[0.25, 0.35]	[0.25, 0.45]	[0.02, 0.20]	[0.20, 0.40]
	S <sub>1</sub>	45	M	VG	G
	S <sub>2</sub>	42	G	VG	M
	S <sub>3</sub>	30	VP	P	G
	S <sub>4</sub>	42	P	G	M
	S <sub>5</sub>	55	VG	VG	G
D <sub>2</sub> [0.40, 0.55]	Weight	[0.20, 0.40]	[0.30, 0.50]	[0.15, 0.20]	[0.25, 0.45]
	S <sub>1</sub>	45	VG	G	G
	S <sub>2</sub>	42	G	M	M
	S <sub>3</sub>	30	M	P	G
	S <sub>4</sub>	42	P	M	M
	S <sub>5</sub>	55	VG	M	G
D <sub>3</sub> [0.60, 0.80]	Weight	[0.15, 0.35]	[0.20, 0.35]	[0.15, 0.25]	[0.35, 0.65]
	S <sub>1</sub>	45	VG	M	VG
	S <sub>2</sub>	42	G	VG	G
	S <sub>3</sub>	30	P	VP	M
	S <sub>4</sub>	42	M	G	P
	S <sub>5</sub>	55	VG	VG	VG

Table 5: Data for national level supplier selection for Lock type B:

	Performance	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>
D <sub>1</sub> [0.10, 0.25]	Weight	[0.25, 0.35]	[0.25, 0.45]	[0.02, 0.20]	[0.20, 0.40]
	S <sub>1</sub>	65	M	G	VG
	S <sub>2</sub>	70	M	G	VG
	S <sub>3</sub>	75	G	M	G
	S <sub>4</sub>	80	VG	VP	M
	S <sub>5</sub>	60	P	M	G
D <sub>2</sub> [0.40, 0.55]	Weight	[0.20, 0.40]	[0.30, 0.50]	[0.15, 0.20]	[0.25, 0.45]
	S <sub>1</sub>	65	G	VG	M
	S <sub>2</sub>	70	G	VG	VG
	S <sub>3</sub>	75	VG	G	G
	S <sub>4</sub>	80	M	P	M
	S <sub>5</sub>	60	VP	G	VP
D <sub>3</sub> [0.60, 0.80]	Weight	[0.15, 0.35]	[0.20, 0.35]	[0.15, 0.25]	[0.35, 0.65]
	S <sub>1</sub>	65	VG	VG	M
	S <sub>2</sub>	70	G	VG	G
	S <sub>3</sub>	75	VP	VG	VG
	S <sub>4</sub>	80	M	P	P

	S <sub>5</sub>	60	M	VP	M
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Crisp weight for lock type A&B can be obtained as

$$W_1 = \frac{0.25 + 0.35}{(0.25 + 0.35) + (0.25 + 0.45) + (0.02 + 0.20) + (0.20 + 0.40)} = 0.28$$

$$W_2 = \frac{0.25 + 0.45}{(0.25 + 0.35) + (0.25 + 0.45) + (0.02 + 0.20) + (0.20 + 0.40)} = 0.33$$

$$W_3 = \frac{0.02 + 0.20}{(0.25 + 0.35) + (0.25 + 0.45) + (0.02 + 0.20) + (0.20 + 0.40)} = 0.10$$

$$W_4 = \frac{0.20 + 0.40}{(0.25 + 0.35) + (0.25 + 0.45) + (0.02 + 0.20) + (0.20 + 0.40)} = 0.28$$

Using the same method, interval weights of DM's importance for Lock type A&B can be obtained as,  $WD_1 = 0.13, WD_2 = 0.35, WD_3 = 0.52$ .

Step III of model algorithm is used to obtain graded mean integration.

The weights of criteria and decision maker's weights are combined using step V of model algorithm.

$$\text{Combined weight} = w_{jk} * WD_j, \quad j = 1, 2, 3. \quad k = 1, 2, 3, 4.$$

Table 6: For lock type A.

	Performance	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>
D <sub>1</sub>	Weight	0.0364	0.0429	0.013	0.0364
	S <sub>1</sub>	45	0.6	0.9	0.8
	S <sub>2</sub>	42	0.8	0.9	0.6
	S <sub>3</sub>	30	0.1	0.33	0.8
	S <sub>4</sub>	42	0.33	0.8	0.6
	S <sub>5</sub>	55	0.9	0.9	0.8
D <sub>2</sub>	Weight	0.084	0.1155	0.049	0.1015
	S <sub>1</sub>	45	0.9	0.8	0.8
	S <sub>2</sub>	42	0.8	0.6	0.6
	S <sub>3</sub>	30	0.6	0.33	0.8
	S <sub>4</sub>	42	0.33	0.6	0.6
	S <sub>5</sub>	55	0.9	0.6	0.8
D <sub>3</sub>	Weight	0.1092	0.1144	0.0832	0.2132
	S <sub>1</sub>	45	0.9	0.6	0.9
	S <sub>2</sub>	42	0.8	0.9	0.8
	S <sub>3</sub>	30	0.33	0.1	0.6
	S <sub>4</sub>	42	0.6	0.8	0.33
	S <sub>5</sub>	55	0.9	0.9	0.9

The weights of criteria and decision maker's weights are combined using step V of model algorithm.

$$\text{Combined weight} = w_{jk} * WD_j, \quad j = 1, 2, 3. \quad k = 1, 2, 3, 4.$$

Table 7: For lock type B.

	Performance	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>
D <sub>1</sub>	Weight	0.0364	0.0429	0.013	0.0364
	S <sub>1</sub>	65	0.6	0.8	0.9
	S <sub>2</sub>	70	0.6	0.8	0.9
	S <sub>3</sub>	75	0.8	0.6	0.8
	S <sub>4</sub>	80	0.9	0.1	0.6
	S <sub>5</sub>	60	0.33	0.6	0.8
D <sub>2</sub>	Weight	0.084	0.1155	0.049	0.1015
	S <sub>1</sub>	65	0.8	0.9	0.6

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D <sub>3</sub>	S <sub>2</sub>	70	0.8	0.9	0.9
	S <sub>3</sub>	75	0.9	0.8	0.8
	S <sub>4</sub>	80	0.6	0.33	0.6
	S <sub>5</sub>	60	0.1	0.8	0.1
	Weight	0.1092	0.1144	0.0832	0.2132
	S <sub>1</sub>	65	0.9	0.9	0.6
	S <sub>2</sub>	70	0.8	0.9	0.8
	S <sub>3</sub>	75	0.1	0.9	0.9
	S <sub>4</sub>	80	0.6	0.33	0.33
	S <sub>5</sub>	60	0.6	0.1	0.6

Table 8: The weighted average of all the criteria for each DMs for lock type A.

Performance	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>
S <sub>1</sub>	45	0.85	0.69	0.86
S <sub>2</sub>	42	0.80	0.80	0.72
S <sub>3</sub>	30	0.41	0.20	0.68
S <sub>4</sub>	42	0.44	0.73	0.44
S <sub>5</sub>	55	0.90	0.80	0.86

Table 9: The weighted average of all the criteria for each DMs for lock type B.

Performance	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>
S <sub>1</sub>	65	0.81	0.89	0.63
S <sub>2</sub>	70	0.77	0.89	0.84
S <sub>3</sub>	75	0.55	0.84	0.86
S <sub>4</sub>	80	0.65	0.31	0.44
S <sub>5</sub>	60	0.35	0.38	0.48

The multi-objective linear programming problem formulation as shown in Appendix A:  
Solve the multi-objective linear programming problem for each objective to get the lower and upper bound.

Table 10:

Objective Function	Lower bound	Upper bound	Tolerance
Cost	270,000	310,350	40,350
Quality	2739.5	3957	1217.5
On time delivery	2810.5	4046.5	1236
Long term relationship	2836	3813	977

Formulate the crisp LPP and maximize the  $\lambda$  subject to all the constraints as well as all the objectives as shown in Appendix B and solve using LINGO software.

Table 11: The results of compensatory model

	$\gamma=1$	$\gamma=0.8$	$\gamma=0.6$	$\gamma=0.4$	$\gamma=0.2$	$\gamma=0$
x <sub>11</sub>	800	800	800	800	800	800
x <sub>21</sub>	700	700	700	700	700	700
x <sub>31</sub>	188	392	499	500	500	0
x <sub>41</sub>	108	0	0	0	0	0
x <sub>51</sub>	204	108	1	0	0	500
x <sub>12</sub>	1200	1200	1200	1200	1200	1200
x <sub>22</sub>	1000	1000	1000	1000	1000	1000
x <sub>32</sub>	1	249	453	455	500	500
x <sub>42</sub>	0	0	0	0	0	0
x <sub>52</sub>	799	551	347	345	300	300
$\lambda$	0.6824	0.6819	0.6723	0.6722	0.656	0
$\mu_{and}$	0.6824	0.6882	0.695	0.7068	0.7199	0.7926
$\lambda_1$	0	0	0	0	0	0.3457
$\lambda_2$	0	0	0	0	0.024	0.8809
$\lambda_3$	0	0	0	0.033	0.067	0.9656
$\lambda_4$	0	0.1268	0.1959	0.1966	0.2309	0.9785

Fig. 4. Aspiration level at different compensatory operator values

In this study, our main objective was to give a compensatory fuzzy method for multi-objective supplier selection model to select suppliers for each product and determine how much quantity should be purchased from each selected suppliers as shown in Table 11 at different grade of compensation. Fig.4 shows the relationship between the grade of compensation and different aspiration of objectives. As the value of  $\gamma$  increases and reaches to 1. The operator  $\mu_{and}$  and  $\lambda$  was coincided and individual aspiration of goals reduced to zero.

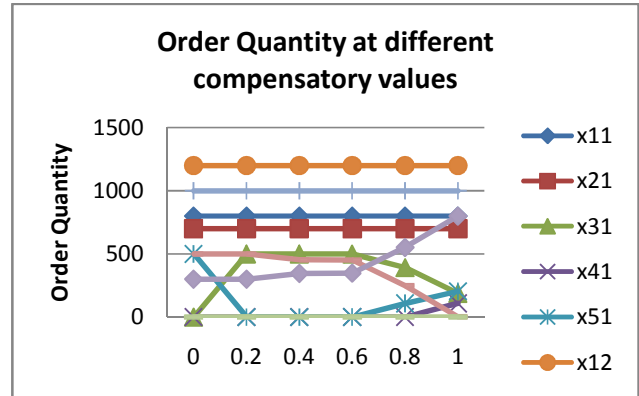
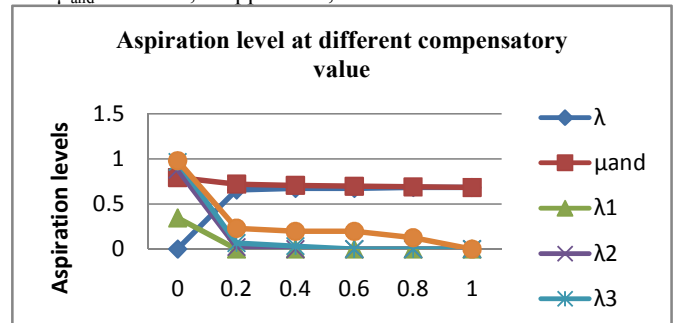


Fig.5. Order quantity at different compensatory operator values

In multi-objective supplier selection problem, cost criterion was qualitative in nature. Quality, on-time delivery and long-term relationship criteria were qualitative in nature. It would be converted into quantitative criteria by using triangular fuzzy numbers. Triangular fuzzy numbers were averaged by using graded mean integration representation method. Also, importance weight and decision maker's (DM's) weight were in intervals. It would be converted into crisp weights and combined to get weight sum one. Finally weighted average of criteria obtained. Multi-objective linear programming problem was formulated and solved. Werner's "fuzzy and" operator used to convert it into crisp formulation at different compensatory values were expressed in Table 11, Fig.4 and Fig.5. It revealed that supplier 1 and supplier 2 were selected and utilized with full capacity for both item at any value of compensatory operator. For  $\gamma=0$  and  $\mu_{and} = 0.7926$ ; supplier 5 were selected for lock type A item having full capacity and lock type B item was 300. Supplier 3 was selected with order quantity 500 for lock type B item only. It means that DM's were satisfied with aspiration  $\approx 80\%$ . For  $\gamma=1$  and  $\mu_{and} = 0.6824$ ; Suppliers 3, 4 and % were selected



with smaller order quantity for lock type A item and only supplier 5 was selected with order quantity 799. The choice of compensation ( $\gamma$ ) would depend on DM's.

Several researchers studied the supplier selection problem [Ozkok et al. 2011] but this modeling approach is unique in the sense that qualitative criteria and interval weights are converted into quantitative criteria and crisp weight respectively. Then using multi-objective supplier selection problem modelled subject to same constraints, such as demand from manufacturers, capacity and budget available at the end suppliers. This model not only selects the suppliers but also gives us idea about how much quantity to be ordered.

### V. CONCLUSIONS

In this study, our main objective was to convert the qualitative criteria into a quantitative criteria and interval weights into the crisp weights. Also to give a compensatory fuzzy method for multi-objective supplier selection problem to select the suppliers for each item and to determine how many items should be purchased from each selected suppliers. Using Werner's "fuzzy and" operator for solving this problem, it gave us strongly efficient solutions. It depends on compensation parameter  $\gamma$  which reflects DM's preferences.

In real cases, many input data are not known precisely for decision making. In this model, imprecise nature of data and varying importance of quantitative and qualitative criteria are considered. The weights of criteria are decided by using intervals. In real cases, the proposed model would be beneficial to DM for finding out the appropriate order quantity to each selected supplier and allows supply chain manager(s) to manage supply chain performance on cost, quality, on time delivery and long term relationship.

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### Appendix A

The multi-objective linear programming problem formulation as follows:

$$\begin{aligned} \text{Min } Z_1 &= 45*x_{11} + 42*x_{21} + 30*x_{31} + 42*x_{41} + 55*x_{51} + 65*x_{12} + 70*x_{22} + 75*x_{32} \\ &+ 80*x_{42} + 60*x_{52}; \\ \text{Max } Z_2 &= \\ &0.85*x_{11} + 0.80*x_{21} + 0.41*x_{31} + 0.44*x_{41} + 0.90*x_{51} + 0.81*x_{12} + 0.77*x_{22} + 0.5 \\ &5*x_{32} + 0.65*x_{42} + 0.35*x_{52}; \end{aligned}$$

## Fuzzy Multi-Objective Supplier Selection Problem for Multiple Items in a Supply Chain

Max  $Z_3$   
 $= 0.69x_{11} + 0.80x_{21} + 0.20x_{31} + 0.73x_{41} + 0.80x_{51} + 0.89x_{12} + 0.89x_{22} + 0.84x_{32} + 0.31x_{42} + 0.38x_{52};$   
 Max  $Z_4$   
 $= 0.86x_{11} + 0.72x_{21} + 0.68x_{31} + 0.44x_{41} + 0.86x_{51} + 0.63x_{12} + 0.84x_{22} + 0.86x_{32} + 0.44x_{42} + 0.48x_{52};$   
 Subject to Constraints  
 $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 2000;$   
 $x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 3000;$   
 $x_{11} \leq 800;$   
 $x_{21} \leq 700;$   
 $x_{31} \leq 750;$   
 $x_{41} \leq 850;$   
 $x_{51} \leq 650;$   
 $x_{12} \leq 1200;$   
 $x_{22} \leq 1000;$   
 $x_{32} \leq 500;$   
 $x_{42} \leq 700;$   
 $x_{52} \leq 900;$   
 $45x_{11} + 65x_{12} \leq 200000;$   
 $42x_{21} + 70x_{22} \leq 300000;$   
 $30x_{31} + 75x_{32} \leq 500000;$   
 $42x_{41} + 80x_{42} \leq 400000;$   
 $55x_{51} + 60x_{52} \leq 200000;$   
 $x_{ij} \geq 0; i = 1, 2, \dots, 5; j = 1, 2. \text{ and integer.}$



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### Appendix B

Formulate the crisp LPP and maximize the  $\lambda$  subject to all the constraints as well as all the objectives.  
 Max  $\lambda + ((1-\gamma)/4) * (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4);$   
 $40350 * (\lambda + \lambda_1) + 45x_{11} + 42x_{21} + 30x_{31} + 42x_{41} + 55x_{51} + 65x_{12} + 70x_{22} + 75x_{32} + 80x_{42} + 60x_{52} \leq 310350;$   
 $-$   
 $1217.5 * (\lambda + \lambda_2) + 0.85x_{11} + 0.80x_{21} + 0.41x_{31} + 0.44x_{41} + 0.90x_{51} + 0.81x_{12} + 0.77x_{22} + 0.55x_{32} + 0.65x_{42} + 0.35x_{52} \geq 2739.5;$   
 $-1236 * (\lambda + \lambda_3) + 0.69x_{11} + 0.80x_{21} + 0.20x_{31} + 0.73x_{41} + 0.80x_{51} + 0.89x_{12} + 0.89x_{22} + 0.84x_{32} + 0.31x_{42} + 0.38x_{52} \geq 2810.5;$   
 $-977 * (\lambda + \lambda_4) + 0.86x_{11} + 0.72x_{21} + 0.68x_{31} + 0.44x_{41} + 0.86x_{51} + 0.63x_{12} + 0.84x_{22} + 0.86x_{32} + 0.44x_{42} + 0.48x_{52} \geq 2836;$   
 $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 2000;$   
 $x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 3000;$   
 $x_{11} \leq 800;$   
 $x_{21} \leq 700;$   
 $x_{31} \leq 750;$   
 $x_{41} \leq 850;$   
 $x_{51} \leq 650;$   
 $x_{12} \leq 1200;$   
 $x_{22} \leq 1000;$   
 $x_{32} \leq 500;$   
 $x_{42} \leq 700;$   
 $x_{52} \leq 900;$   
 $45x_{11} + 65x_{12} \leq 200000;$   
 $42x_{21} + 70x_{22} \leq 300000;$   
 $30x_{31} + 75x_{32} \leq 500000;$   
 $42x_{41} + 80x_{42} \leq 400000;$   
 $55x_{51} + 60x_{52} \leq 200000;$   
 $\lambda + \lambda_1 \leq 1;$   
 $\lambda + \lambda_2 \leq 1;$   
 $\lambda + \lambda_3 \leq 1;$   
 $\lambda + \lambda_4 \leq 1;$   
 $x_{ij} \geq 0; i = 1, 2, \dots, 5; j = 1, 2. \text{ and integer.}$