Multi-Objective Transportation Problem Under Fuzziness with S-type Membership Function

M. A. MUJEEB KHAN, SYED JAVED KABEER

Abstract— **Transportation problem is a key optimization technique used these days for planning, especially, to minimize the cost. As a fact, in case of emergency, instead of cost the time plays an important role. Several researchers have been carried out research in Bi-objective transportation problem to cope up with such problems arises due to disasters. In such cases timely help and remedies become more important than any other parameters involved in the problem. Uncertainty arises due to unseen and unavoidable factors; such characteristics well formulated using fuzzy logics. Therefore, this paper discusses multi-objective transportation problem under fuzziness with S-type membership function. Problem is formulated and illustrated numerically.**

Index Terms— **Multi-objective Transportation problem, S-type Membership Function, Optimization Problem**

I. INTRODUCTION

The Transportation Problem (TP) was first developed and proposed by F. L. Hitchcock since 1941[1], [2]. It usually aims to minimize the total transportation cost [3]-[7]of items, initially stored at different locations (origins) and need to transport at various destinations. Other objectives that can be set are a minimization of the total delivery time, irrespective of cost, this is more important when an emergency occurs viz; floods, earthquakes etc. Due to flexibility and easiness of TP, a method could be used to a maximization of the profit, etc.This way TP can be used to get the optimal solution by setting more than one objective. Mathematically, a typical transportation problem can be presented as:

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_{i,}\ i = 1,2,\dots\dots,m \quad \text{(Supply)}$$

$$\sum_{i=1}^{m} x_{ij} = b_{j,}\ j = 1,2,\dots\dots,n \quad \text{(Demand)}$$

$$x_{ij} \geq 0 \quad \text{for all } i \text{ and } j \quad \dots(1)$$

Where,

x_{ij} = the amount of goods moved from origin i to destination j

c_{ij} = the cost of moving a unit amount goods from origin i to destination j

a_i = the supply available at each origin i

b_j = the demand available at each destination j

m = total number of origins

n = total number of destinations

This problem can be solved by classical transportation methods.

In reality, considering only one objective of TP is not sufficient because it may not lead to the practical optimal solution. Thus the Decision Maker (DM) is rather to pay attention on several objectives on same time or in other words we can describe this as the limitation of single objective TP. This limitation can be sought out by generating multi-objective TP.

Multi-objective Transportation Problem (MOTP):

The multi-objective transportation model is set to solve the transportation problem simultaneously associated with several objectives. Normally, existing multi-objective transportation models use a minimization of the total cost objective as one of their objectives. The other objectives may concern about delivery time, quantity of goods delivered, under used capacity, reliability of delivery, energy consumption, safety of delivery, etc. The multi-objective transportation problem with k objectives can be represented as follows [8]:

$$\min f^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c^k{}_{ij}x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_{i,}\ i = 1,2,\dots\dots,m \qquad \text{for all } i.$$

$$\sum_{i=1}^{m} x_{ij} = b_{j,}\ j = 1,2,\dots\dots,n \qquad \text{for all } j.$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \text{ and } x_{ij} \geq 0 \text{ for all } i \text{ and } j \quad \dots(2)$$

Where c_{ij}^k represents the coefficients related to x_{ij} variable for objective k.

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In real life situations more than one or two objectives are concerned. The first objective is to minimize the total transportation cost which is the baseline objective for all transportation models. The second objective is to minimize the overall transportation time. The solution of MOTP depends on data related to cost and time from origin and destination which may not always correct due to some factors stated above and hence it is difficult to find actual optimum cost and time in practical world and in this case our transportation problem solution fails.

Multi objective Transportation problem in Fuzzy sense with S-membership function:

Now a day, the fuzzy set theory has been also developed in a large area and its different modification and generalization form have appeared. In many real situations, there are capacity restrictions on units of commodities which are shipped from different sources to different destinations. In the model formulation, supply and demand constraints are converted into equivalent deterministic forms. Then, we define the fuzzy goal levels of the objective functions. The fuzzy objective goals are then characterized by the associated membership functions.

In the last twenty years, the multi-objective transportation problem has been investigated in the sense of fuzzy set theory [8,9,10]. This fuzzy programming technique is more flexible and allows finding the solutions which are more sufficient to the real problem. In fuzzy optimization, the degree of acceptance of objectives and constraints are considered only.

Several researchers has been tried different membership functions like linear membership function, trapezoidal function, hyperbolic membership function. The membership function is so chosen is appropriate as per situation occurs. In this paper an attempt is made to present the situation more appropriate using S-membership function.

Following is graph of S-membership function

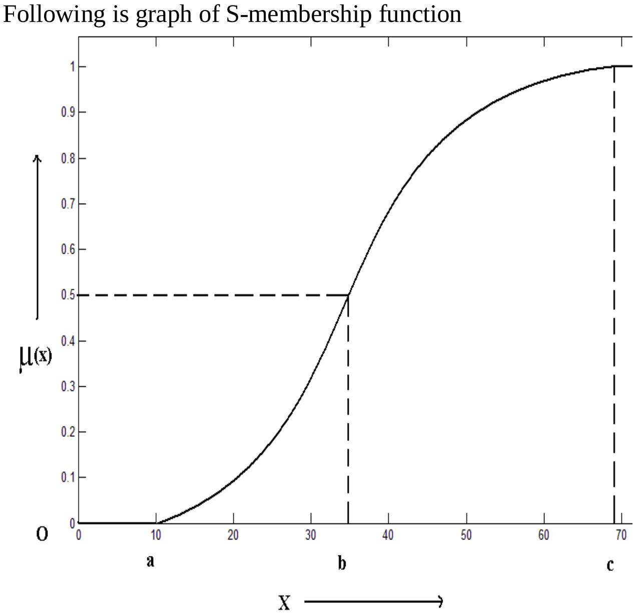

Figure 1: S-membership Function

S-shaped membership function may be represented as**:**

$$\mu_S(x, a, b, c) = \begin{cases} 0, & \text{for } x \le a \\ 2[(x-a)/(c-a)]^2, & \text{for } a \le x \le b \\ 1-2[(x-c)/(c-a)]^2, & \text{for } b < x \le c \\ 1, & \text{for } x \ge c \end{cases}$$

Algorithm:

Step 1: Solve the multiobjective transportation problem as a single objective transportation problem, taking each time only one objective as objective function and ignoring all others.

Step 2: Compute the value of each objective function at each solution derived in Step 1.

Step 3: From Step 2, find for each objective the best-L_k (aspired level of achievement) and the worst-U_k (highest acceptable level of achievement) values corresponding to the set of solutions.

Step 4: Define a membership functions of appropriate fit to situation (in our case S-shaped membership function is chosen).

Step 5: Convert the fuzzy mode of the problem, obtained in step 5, into the following crisp Model;

Maximize λ **....**(1)

Subject to

$$\lambda \le \mu(Z_K)$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1,2,\ldots\ldots, m$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1,2,\ldots\ldots, n$$

$$X_{ij} \ge 0, \quad \lambda \ge 0 \quad \forall i \text{ and } j \ldots.(3)$$

Using S- shaped membership function (3) can be converted into

Maximize λ

Subject to

$$Z_K + L_K + \lambda\left(\frac{1}{2}(U_K - L_K)\right)^{\frac{1}{2}} \le U_K \qquad \text{for } K = 1,2,\ldots\ldots.K$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1,2,\ldots\ldots, m$$

$$\sum_{j=1}^{m} X_{ij} = b_j, \quad j = 1,2,\ldots\ldots, n$$

$$X_{ij} \ge 0, \quad \forall i \text{ and } j$$

Example: The data is collected for supplying to different companies after taking from different sources.
Following are information in tabular form:

Data for time:

	D1	D2	D3	D4	supply
S1	16	19	12	11	16
S2	22	13	9	8	28
S3	14	28	8	7	12
Demand	10	15	17	14	56

Data for cost:

	D1	D2	D3	D4	supply
S1	9	14	12	8	16
S2	16	10	14	9	28
S3	8	20	6	5	12
Demand	10	15	17	14	56

The given problem may be formulated as:

$$Min\ Z_1 = 16x_{11} + 19x_{12} + 12x_{13} + 11x_{14} + 22x_{21} + 13x_{22} + 19x_{23} + 8x_{24} + 14x_{31} + 28x_{32} + 8x_{33} + 7x_{34} \ .....(P1)$$

$$Min\ Z_2 = 9x_{11} + 14x_{12} + 12x_{13} + 8x_{14} + 16x_{21} + 10x_{22} + 14x_{23} + 9x_{24} + 8x_{31} + 20x_{32} + 6x_{33} + 5x_{34} \ .....(P2)$$

and rest all are zeroes.

$$Z_2\left(X^{(2)}\right) = 494 \text{ and } Z_1\left(X^{(2)}\right) = 630$$

Using these results, obtained in step1, the pay-off matrix is
From the pay-off matrix

$$U_1 = \max\{626, 630\} = 630$$

$$U_2 = \max\{494, 497\} = 497$$

$$L_1 = \min\{626, 630\} = 626$$

$$L_2 = \min\{494, 497\} = 494$$

If S-type membership function is used then (P1) and (P2) can be converted into following equations:
(P1 –A)
Maximize λ

subject to

$$16x_{11} + 19x_{12} + 12x_{13} + 11x_{14} + 22x_{21} + 13x_{22} + 19x_{23} + 8x_{24} + 14x_{31} + 28x_{32} + 8x_{33} + 7x_{34} + 1.41\lambda \le 630$$

$$\sum_{j=1}^{4} X_{1j} = 16, \sum_{j=1}^{4} X_{2j} = 28, \sum_{j=1}^{4} X_{3j} = 12$$

$$\sum_{i=1}^{3} X_{i1} = 10, \sum_{i=1}^{3} X_{i2} = 15, \sum_{i=1}^{3} X_{i3} = 17 \sum_{i=1}^{3} X_{i3} = 14$$

$$X_{ij} \ge 0, i = 1,2,3 \quad j = 1,2,3,4$$

Solving the above problem, the optimal solution is:

$$X_{11} = 9, X_{31} = 1\ X_{13} = 7,\ X_{22} = 15, X_{24} = 13,$$
$$X_{33} = 10,\ X_{34} = 1$$
and rest of all zeros

$$Z_1^{(o)} = 623 \text{ and } Z_2^{(o)} = 493,\ \lambda = 0.371$$

(P2 –A)

Maximize λ

subject to

$$9x_{11} + 14x_{12} + 12x_{13} + 8x_{14} + 16x_{21} + 10x_{22} + 14x_{23} + 9x_{24} + 6x_{33} + 5x_{34} + 1.22\lambda \le 497$$

$$\sum_{j=1}^{4} X_{1j} = 16, \sum_{j=1}^{4} X_{2j} = 28, \sum_{j=1}^{4} X_{3j} = 12$$

$$\sum_{i=1}^{3} X_{i1} = 10, \sum_{i=1}^{3} X_{i2} = 15, \sum_{i=1}^{3} X_{i3} = 17,$$

$$\sum_{i=1}^{3} X_{i3} = 14$$

$$X_{ij} \ge 0, i = 1,2,3 \quad j = 1,2,3,4$$

Solving the above problem, the optimal solution is:

$$X_{11} = 10,\ X_{13} = 5, X_{14} = 1, X_{22} = 15,$$
$$X_{24} = 13, X_{33} = 12$$
and rest of all zeros

$$Z_1^{(o)} = 626 \text{ and } Z_2^{(o)} = 490,\ \lambda = 0.194$$

$$Z_2^{(o)} = 491 \text{and } Z_1^{(o)} = 627$$

II. CONCLUSION

In this paper an attempt is made to formulate multi-objective transportation problem using fuzzy membership function especially-S-type membership function. Present paper shows appropriateness of such type membership functions to represent vagueness and uncertainty up to certain extent. This helps in reduction of cost, time and other factors in transportation problem. It also provided that results are better improved in certain cases for the type membership function is used. Study can be further generalized for other case where the above membership function is suitable to apply.

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