Mathematical Modeling with Technology

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Abstract— **Mathematics is very important for an increasing number of technical jobs in a quantitative world. Over recent years technology has progressed to the point where many computer algebraic systems (CAS) are widely available. These technologies can perform a wide variety of algebraic manipulations.Computer algebra systems (CAS) have become powerful and flexible tools to support a variety of mathematical activities. This includes learning and teaching, research and assessment. MATLAB, an acronym for "Matrix Laboratory", is the most extensively used mathematical software in the general sciences. MATLAB is an interactive system for numerical computation and a programmable system. This paper attempts to investigate the impact of integrating mathematical modeling and applications on higher level mathematics in India. In this paper we also establish a link between three computational techniques using MATLAB.**

Index Terms— Computational techniques, Computer Algebraic Systems (CAS), MATLAB, Mathematical Modeling**.**

I. INTRODUCTION

We live in a mathematical world. The level of mathematical thinking and problem solving needed in the workplace has increased dramatically. In such a world, those who understand and can do mathematics will have opportunities that others do not. Mathematical competence opens doors to productive futures. A lack of mathematical competence closes these doors. In this age of Information Technology students need to learn a new set of mathematics basics that enables them to solve problems creatively and resourcefully [8]. Increasingly, many countries across the world have witnessed a growing collection of research on including mathematical applications and modeling in undergraduate level. The benefits of integrating applications and modeling in the mathematics learning are manifold. They help to motivate students for applying mathematical concepts and learn new mathematical content. Research has also shown that the use of technology tools can greatly enhance student's explorations and investigations. Today, advances in computations are linked to the development of numerical and symbolic mathematical software, among which Computer Algebra Systems (CAS) play an increasing role. In the enhancement of technology MATLAB is also widely used in all areas of applied mathematics, in education and research at universities.

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Computer Algebraic System (CAS)

A Computer Algebra system is a type of software package that is used in manipulation of mathematical formulae. The primary goal of a Computer Algebra system is to automate tedious and sometimes difficult algebraic manipulation tasks. Computer Algebra systems often include facilities for graphing equations and provide a programming language for the user to define their own procedures. Computer Algebra systems have not only changed how mathematics is taught at many universities, but have provided a flexible tool for mathematicians worldwide. Examples of popular systems include Maple, Mathematica, and MathCAD. Computer Algebra systems can be used to simplify rational functions, factor polynomials, find the solutions to a system of equation, and various other manipulations. In Calculus, they can be used to find the limit of, symbolically integrate, and differentiate arbitrary equations.

MATLAB

MATLAB is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. It is an interactive system whose basic data element is an array that does not require dimensioning. This allows us to solve many technical computing problems, especially those with matrix and vector formulations, in a fraction of the time. It is not a CAS (computer algebra system) but it has an add-on Symbolic package. MATLAB, in its default mode, works in floating point arithmetic, which is much faster for performing computations with sufficient accuracy for most applications. For instances where symbolic functionality is required, MATLAB offers access to the Maple kernel (which is built in), and thus can offer the best of both worlds. With its superb graphics and user-friendly interface, MATLAB is an ideal software to enhance university mathematics courses. In university environments, it is the standard instructional tool for introductory and advanced courses in mathematics, engineering, and science. In industry, MATLAB is the tool of choice for high-productivity research, development, and analysis. MATLAB features a family of application-specific solutions called toolboxes. Very important to most users of MATLAB, tool boxes allow us to learn and apply specialized technology.

II. RELATED WORK

In recent years research has been done on the relation between computer techniques and conceptual understanding. For instance, attention has been paid to the process of instrumentation, in which conceptual insights and technical skills come together. A link between technical work in a computer algebra environment and conceptual understanding has been seen in university students [3]. A number of research studies have investigated student's

performance on achievement tests that measured computational or procedural skills after mathematics instruction that incorporated CAS. The effective use of CAS requires greater mathematical understanding and concept development. Knowing which operations to choose, which processes to follow and interpreting the result are all higher level cognitive functions than knowing the steps of a manipulative process. CAS is described as mathematical investigative assistant through which students investigate the problems strategically and purposefully[4]. It has been described that Integrating mathematical modeling and applications for teaching led to a very satisfying combination of technology use and 'by-hand' skills. An experimental research has been conducted regarding the effect of introducing scientific software into the learning experience upon attitudes and the learning process. The research aimed at investigating the effect of using MATLAB software in building attitudes and using strategies for learning Geometry with the help of the computer. It was appreciated that the relevance of the efficient use of the computer in enhancing the motivational potential, in relation to learning Mathematics and suggested that the graduates from the specialization of Mathematics should build the skills needed to exploit the available technological resources [1].

III. PROPOSED SYSTEM

Computational skills are the selection and application of arithmetic operations to calculate solutions to mathematical problems. Research shows that a person fluent with a particular Mathematics concept or principle has three distinctive and defining characteristics:

• They can decide when it might be useful to use this concept or principle;

• They have the symbolic skills to do it correctly;

• They know what the significance of the end product is.

It is suggested that too much time has been spent traditionally on the second of these and too little time

on the first and last [8]. In this paper we establish a link amongst three computational techniques by using MATLAB. These techniques are explained in figure1. Conceptual computational technique is a traditional technique in which we compute the mathematical problems by hand without using computer. The other two techniques are based on computer software's like CAS and MATLAB. In representational technique student can introduce parameters, use realistic data and see the result of changes in parameters and data immediately. This often leads to interesting discussions of mathematical models depending on these parameters.
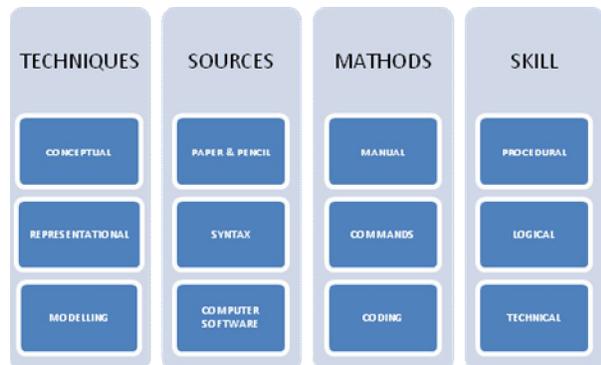
IV. EXPERIMENTAL SET UP

The research was conducted at the Sadabai Raisoni Women's College, Nagpur in India and consisted in assisting and observing 8 lessons of Mathematics and 16 lessons of Information and Communication Technologies, involving a group of 30 students. During a period of twelve weeks the students attended four 2-hour lessons each week. Four of these weekly lessons were taught in a more traditional way to group of 30 students (Conceptual technique), in which alternately the teacher was explaining the mathematics and the students were studying the subject, for instance by making assignments, but where no computer facilities were available. In the next four 2-hour weekly lessons a computer algebra environment on a PC (Matlab 7.14) was used to taught the syntaxes related to topic(Representational technique). In the last four 2-hour weekly lesson again the computer algebra environment on a PC (Matlab 7.14) was used to teach the programming related to topic (Modelling technique). Last two sections include a detailed description of the laboratory module in which the students explored the particular topic using MATLAB. The aim is to highlight the role of MATLAB in enabling the learning process through these sessions. MATLAB helped students to verify their paper and pencil work, visualise symbolic expressions graphically, explore functions numerically and make conjectures leading to their understanding the concept.

An illustration

We will illustrate here how we have worked out this structure in an example from Calculus: Computing the Laplace Transform. In the first session through 'traditional' lessons, the group of 30 students has learned how to find Laplace and inverse Laplace Transform of a function by conceptual technique. In this technique they have learnt how to compute the Laplace transforms by paper and pencil method. In next session these students have learnt the same topic using MATLAB, syntaxes and commands by representational technique. And in the last session the student learn MATLAB programming for computing the Laplace Transforms by modeling technique.

First, the students have to solve some exercises with paper-and-pencil. Subsequently, we let them solve the system with MATLAB commands and functions, and compare the answer with the result they had obtained by hand. After completing all sessions, we have conducted a test related to topic. Students have to compute the questions manually by hand in first test. In next two tests students have to solve the questions using Matlab commands and functions practically in computer laboratory.

Consider the following set of questions:

i) $L(e^{2t} sin3t) = \frac{3}{(s-2)^2+3^2}$

ii) $L^{-1}(\frac{1}{(s-2)(s+4)} + \frac{1}{s+5}) = e^{-5t} - 2e^{-4t} + 1$

Session 2: students solved the exercise by MATLAB commands

Session 3: students solved the exercise by MATLAB functions

V. ANALYSIS AND RESULTS

A range of studies showed that the overall conceptual understandings of the students who used a CAS were the same as or better than the understandings of their counterparts who did not use a CAS. Other studies showed that students who used a CAS better understood particular concepts – variable and function – than those who did not use a CAS. From these studies, we conclude that CAS use enables students to develop deeper conceptual understanding".

In this research, we have compared the performance of the students in every test which is conducted after completing the Sessions. We accomplish this comparison by statistical analysis. Table 1, shows the mean and standard deviation for scores obtained by the students in various sessions tests. The highest mean scores were in session-3 (8.43±0.50) with the range of 5-8 scores. In session -1 the mean score was 6.63± 0.76, which is the least mean score obtained by the students among all sessions. For testing the hypothesis the mean scores obtained by students in three sessions were analyzed statistically to determine whether there was a significant difference in the mean scores. The data was evaluated using Analysis of Variance (ANOVA) statistical method and the result obtained is presented in table 2.

Statistical Analysis of Performance in different Sessions								
Sessions	N	Mean	Std. Devi-ation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Session1	30	6.6333	0.7649	0.1397	6.3477	6.9189	5	8
Session2	30	8.1333	0.7303	0.1333	7.8606	8.4060	7	9
Session3	30	8.4333	0.5040	0.0920	8.2451	8.6215	8	9
Total	90	7.7333	1.0364	0.1093	7.5163	7.9504	5	9

Table 1.

ANOVA (Analysis of Variants)					
Performance					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	55.8	2	27.9	60.987	0
Within Groups	39.8	87	0.457		
Total	95.6	89			

Table 2.

The analysis of results indicates that the mean scores obtained by the students in different sessions test differ significantly. We observe that the student's performance has been improved in MATLAB included sessions.

VI. CONCLUSION AND FUTURE WORK

The proposed all three computational skills are important but students performed better in representational and modelling technique as compared to conceptual technique. We can conclude that the students will be able to model a mathematical problem using MATAB and demonstrate the ability to model mathematical problems through the use of technology. We strongly recommend the use of MATLAB in higher level mathematics education in India.

In future work, we can enhance the features of technology which would support the learning of undergraduate mathematics, in terms of computation, reasoning, and visualization. We can increase the use of contemporary software to meet the needs of undergraduate practice.

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