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Semi-Regular Group Divisible Designs For Smaller Block Size

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Abstract: **A group divisible (GD) design is said to be Singular (S) if $r - \lambda_1 = 0$; Semi regular (SR) if $r - \lambda_1 > 0$ and $rk - v\lambda_2 = 0$; Regular (R) if $r - \lambda_1 > 0$ and $rk - v\lambda_2 > 0$. In the paper, a new procedure of constructing SRGD design with $k = m$ and $\lambda_1 = 0$, is proposed from a parent SRGD after reducing number of treatment and same number of blocks without disturbing its Semi-Regularity property. It privileges the experimenters to decrease the number of treatment without affecting the number of blocks. Such designs are useful in civil engineers, Agricultural experiments and others.**

AMS Subject Classification: 05 B05

Key Words:**Semi-Regular Group Divisible design, Balanced Incomplete Block design, orthogonal array**.

I. INTRODUCTION

The methods of constructing resolvable group divisible designs are available in many literatures. Meitei [1] has constructed an increase of block-size of SRGD from the parent SRGD design by 1, making an increase of treatment numbers of the parent SRGD design, by n. Dey and Nigam [10] have suggested a construction method of GD design with parameters ($v=v^*/s$, $b= b^*/s$, $r=r^*$, $k=k^*$, $\lambda_1=0$, $\lambda_2= s\lambda_2$; $m=m^*$, $n=t$), starting from another GD designs with parameters $v^*=m^* n^*$, b^*, r^*, k^*, $\lambda_1^*=0$, λ_2^*; m^*, $n^*=st$ ($s\geq 2$, $t\geq 2$). If the starting design is SRGD i.e. $r^*-\lambda_1^*>0$ and $r^*k^*-v^*\lambda_2^*=0$ i.e., $r^*>\lambda_1^*$ and $r^*k^*-m^*n^*\lambda_2^*=0$, the resultant design remains as Semi-regular.

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Many relations among the parameters of the design have been proposed by Bose and Connor [4] for the existence of Group Divisible (GD). Using combinatorial methods and the orthogonal arrays, many series of GD designs are presented in the literature of Bose, Shrikhande and Bhattacharya [5] and Raghava Rao [7]. Kageyama and Tsuji [9] have proposed (i) a GD design is Semi-Regular iff k/m is an integer and every block contains exactly k/m treatment(s) from each group of the association scheme and (ii) a GD design is singular iff k/n is integer and every block contains exactly k/n groups of the association scheme. Kageyama [11] has constructed α-resolvable regular GD design starting from α-resolvable Balanced Incomplete Block (BIB) design.

Orthogonal Array: A $k \times n$ matrix B with entries from a set S, containing $s(\geq 2)$ symbols is said to be an orthogonal array with k constraints, N assemblies, s symbols and strength t if every $t \times N$ sub-matrix of B contains each of s^t t-plats of the s symbols as columns, equally often, say, λ times each where $t \leq k$. The integer λ is called the index of the array. It is denoted by $A(N, k, s, t)$. There are s^t possible t-plats of elements from S having s element. Each possible s^t t-plats occurs λ times in every $t \times N$ sub-matrix of B. Thus $N = \lambda s^t$.

By the definition, it is obvious that the existence of $A(N, k, s, t)$ implies that of $A(N, k, s, t_1)$ where $t_1 < t$. However its index will change accordingly.

II. CONSTRUCTION

Let us consider a SRGD design, D with parameters $v = mn,\ b = n^2\lambda_2,\ r = n\lambda_2,\ k = m,\ \lambda_1 = 0,\ \lambda_2;\ m,\ n.$ Further, let $n(i-1)+j$ be the j^{th} treatment in the i^{th} group of the association scheme; $i = 1, \dots, m;\ j = 1, \dots, n$. Since k is divisible by m $(i.e.\ k/m = 1)$ Kageyama and Tsuji [9], every block of D contains exactly one element from each group. Let us define m groups of the Group Divisible association scheme of D, as given by $G_i = \{(i-1)n+0, (i-1)n+1, \dots, (i-1)n+n-1\}$ for $i = 1, \dots, m$. The block of SRGD design D is written as columns such that the element i^{th} group is placed in the i^{th} row. That is, the blocks structure is such that each of all the 1^{st} positions of the blocks are occupied by each of the elements from the 1st group; that of the 2nd positions by that of the elements from the 2nd group, so on. Replacing the treatment $(i-1)n+\theta$ of the i^{th} group by θ, for $\theta = 0, 1, \dots, n-1$, we get an orthogonal array, $A[n^2\lambda_2, m, n, 2]$, due to Bose *et al* [5].

Deleting the last $m - m_1$ rows from the orthogonal array A without disturbing the 1st m_1 rows of A, the parent orthogonal array gives A^*, say, with $n^2\lambda_2$ assemblies, m_1 $(< m)$ constraints, n symbols and strength 2 viz;

$A^*=[n^2\lambda_2, m_2, n_2]$ … (2.1)

Again, replacing the element θ in the l^{th} row of the array A^* with an element given by

$(l-1)n + \theta$; $l=1, \dots, m_1 (< m)$, $\theta = 0, 1, \dots, n-1$

i.e., θ in the l^{th} row of $A^* \to (l-1)n + \theta$ … (2.2)

The columns of the array A*, given by the relation (2.2), are the blocks of a resultant SRGD. Considering $G_1, \dots, G_{m_1}$ of the parent association scheme as the m_1 groups of a resultant design, we can get the following theorem.

A.Theorem: The existence of a SRGD design with parameters $v = mn$, $b = n^2\lambda_2$, $r = n\lambda_2$, $k = m$, $\lambda_1 = 0$, λ_2; m, n implies that of an SRGD design with parameters $v^* = m_1 n$, $b^* = n^2\lambda_2$, $r^* = n\lambda_2$, $k^* = m_1 (< m)$, $\lambda_1^* = 0$, λ_2^*; $m^* = m_1$, $n^* = n_1$.

Proof:- The resultant orthogonal array A^* given in the (2.1), is with $n^2\lambda_2$ assemblies, m_1 constraints, n symbols. By the method of construction given in the relation (2.2), there are $n^2\lambda_2$ blocks (i.e., $b^* = n^2\lambda_2$) of size m_1 each (i.e., $k^* = m_1$). Further, each of m_1 rows of the resultant orthogonal array A^* contains n symbols. Moreover, each element in each row of A^* produces distinct element as $l = 1, \dots, m_1 (< m)$; $\theta = 0, 1, \dots, n-1$ and the number of distinct elements of the resultant design is nm_1 i,e., $v^* = nm_1$. As the strength of the orthogonal array A^* is t (≥ 2), every $2 \times n^2\lambda_2$ sub-matrix of the orthogonal array A^* contains each of n^2 2-plats {viz; $(x_1, x_2)'$; $x_1, x_2 \in \{0, 1, \dots, n-1\}$} of the n symbols viz; $0, 1, \dots, n-1$ as column, equally often, λ_2 times [as $N = \lambda n^t$]. Then any symbols θ $(i.e.\ 0, 1, \dots, n-1)$, every pair of θ with each element of $\{0, 1, \dots, n-1\}$ in column form appears λ_2 times in A^*. Then θ appears $n\lambda_2$ times.

Any two treatments $(l-1)n + \psi$ and $(l'-1)n + \phi$; $(l \neq l')$; $\psi, \phi = 0, 1, \dots, n-1$ of the resultant design are produced from the l^{th} and l'^{th} rows of A^* respectively. Consequently, $(l-1)n + \psi$ and $(l'-1)n + \phi$ belong to G_l and $G_{l'}$ respectively by the relation (2.2) and are second associates to one another. Let us count in how many blocks of the resultant design they occur together. Any two treatments (ψ, ϕ) in A^* occurs λ_2 times (in column form) i.e., $(\psi, \phi)'$ in the sub-matrix $2 \times n^2\lambda_2$ of A and accordingly of A^* also. As the index of an orthogonal array does not change when some rows of the concerned orthogonal array are removed. The $(l-1)n + \psi$ and $(l'-1)n + \phi$ occur together in λ_2 blocks by the relation (2.2). So, $\lambda_2^* = \lambda_2$. By the relation (2.2) the l^{th} positions of any block of the resultant design are occupied by a single element of G_l $(l = 0, 1, \dots, m_1)$. Thus no block of the resultant design contains any two elements belonged to the same group of the association scheme. Therefore $\lambda_1^* = 0$.

Using an isomorphic solution of SRGD design SR36, Clathworthy [6], an illustration is made below.

A. Example: The blocks of the isomorphic solution of SRGD design SR36 design with the parameters $v = 8$, $b = 8$, $r = 4$, $k = 4$, $n = 2$, $\lambda_1 = 0$, $\lambda_2 = 2$, $m = 4$, $n = 2$ are $B_1 = \{0, 2, 4, 6\}$, $B_2 = \{1, 3, 5, 7\}$, $B_3 = \{2, 5, 7, 0\}$, $B_4 = \{3, 4, 6, 1\}$, $B_5 = \{4, 7, 0, 3\}$, $B_6 = \{5, 6, 1, 2\}$, $B_7 = \{6, 0, 3, 5\}$, $B_8 = \{7, 1, 2, 4\}$

based on the association scheme with 4 groups, each containing 2 elements: $G_1 = \{0, 1\}, G_2 = \{2, 3\}, G_3 = \{4, 5\}$ and $G_4 = \{6, 7\}$. By the construction method of new blocks of the resultant SRGD design and the elements are arranged such that the 1st position is occupied by the element from 1st group, 2nd position by 2nd group and so on. The new blocks becomes

$B_1 = \{0, 2, 4, 6\}$, $B_2 = \{1, 3, 5, 7\}$, $B_3 = \{0, 2, 5, 7\}$, $B_4 = \{1, 3, 4, 6\}$, $B_5 = \{0, 3, 4, 7\}$, $B_6 = \{1, 2, 5, 6\}$, $B_7 = \{0, 3, 5, 6\}$, $B_8 = \{1, 2, 4, 7\}$.

By replacing the treatment $(i-1)n + \theta$ of the i^{th} group by θ for $\theta = 0, 1, \dots, n-1$, we have

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$A^* = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$ after deleting the last row of orthogonal array A.

Rewriting the treatment θ in the l^{th} row of A^* by $(l-1)2 + \theta$; $l = 1,2,3$ and $\theta = 0,1$ yields the 8 blocks and the 3 groups of the required SRGD design $v^* = 6$, $b^* = 8, r^* = 4, k^* = 3, \lambda_1^* = 0, \lambda_2^* = 2, m^* = 3, n^* = 2$ as given $B_1 = \{0, 2, 4\}$, $B_2 = \{1, 3, 5\}$, $B_3 = \{0, 2, 5\}$, $B_4 = \{1, 3, 4\}$, $B_5 = \{0, 3, 4\}$, $B_6 = \{1, 2, 5\}$, $B_7 = \{0,3,5\}$, $B_8 = \{1, 2, 4\}$ and $G_1 = \{0, 1\}, G_2 = \{2, 3\}, G_3 = \{4, 5\}$ respectively.

III. CONCLUSION

In agricultural experiment when the experimental area, where a SRGD is to be applied, could not provide the sufficient block size, by using such proposed construction-method of SRGD it can reduce the block size.

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