MINIMIZING RENTAL COST IN THREE-MACHINE FLOWSHOP PROBLEMS

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***Abstract*— This paper studies three-machine specially structured flow-shop problems under rental Policy III. Here under Policy III, it is considered that first machine will be taken on rent in the starting of processing the jobs; second machine will be taken on rent when the first job is completed on first machine and third machine is taken on rent when first job is completed on second machine. The objective is to obtain a sequence which minimizes the total rental cost of the machines. An algorithm is developed using Branch-and-Bound technique. The algorithm is illustrated through a numerical example.**

***Index Terms*— Flow-shop scheduling, Elapsed Time, Idle Time Rental Cost**

I. INTRODUCTION

In flow-shop sequencing problem, when one has got the assignment but does not have one's own machines or does not have enough money or does not want to take risk of investing money for the purchase of machines, under these circumstances, may take machines on rent to complete the assignment. Minimization of total rental cost of machines will be the criterion in these types of situations.

The following renting policies generally exist:

***Policy I**: All the machines are taken on rent at one time and are returned also at one time.*

***Policy II**: All the machines are taken on rent at one time and are returned as and when they are no longer required.*

***Policy III**: All the machines are taken on rent as and when they are required and are returned as and when they are no longer required for processing.*

Various authors [1-7] studied these rental policies to optimize the given objective function. Under ***Policy I***; the sequence which minimizes the total elapsed time will also minimizes the total rental cost of the machines. For 2-machine problem Johnson [8]; for 3-machine problem Lomnicki [10] and Bagga [7]; and for m-machine problem Gupta [2] and Pawalker and Khan [9] can be applied to minimize the total rental cost of machines. Under ***Policy II***; in case of 2-machine flow-shop problem, Johnson [8] will provide the optimal sequence. For 3-machine flow-shop problem, Bagga and Ambika [5] provided a Branch-and-Bound algorithm. Bagga and Khurana [6] use Branch-and-Bound technique to solve n-job, 2-machine flow-shop problems under two policies: (i) when both machines are hired simultaneously and (ii) when the 2nd machine is hired only when the first job is completed on 1st machine.

In this Paper, ***Policy III*** is adopted for three-machine flow-shop problems. Under ***Policy III***, it is considered that first machine will be taken on rent in the starting of processing the jobs; second machine will be taken on rent when the first job is completed on first machine and third machine is taken on rent when first job is completed on second machine. The objective is to obtain a sequence which minimizes the total rental cost of the machines

An algorithm is developed using Branch-and-Bound technique. Numerical example is also given to demonstrate the algorithm.

II. MATHEMATICAL FORMULATION

A. *Notations:*

- S : Sequence of jobs 1, 2, …, n.
- M_j : Machine j, $j = 1, 2, 3$.
- p_{ij} : Processing time for job i on machine M_j.
- I_{ij} : Idle time of machine M_j for job i.
- C_j : Rental cost per unit time of machine M_j, $j = 1, 2, 3$.
- J_r : Partial schedule of r scheduled jobs.
- J_r' : Set of remaining (n-r) free jobs.
- i_1 : Job at 1st position of partial schedule J_r.
- $Z_{ij}(S)$: Completion time of i^{th} job of sequence S on machine M_j.
- $Z_{i_1,j}(S)$: Completion time of job i_1 of sequence S on machine M_j.
- $t(J_r, j)$: Time when the last job of the assigned schedule J_r is completed on machine M_j, $j = 1, 2, 3$.

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LB[Jr] : Lowest possible rental cost corresponding to partial schedule J_r, irrespective of any schedule of J_r'.

Let n jobs require processing over three machines M_1, M_2 and M_3 in the order $M_1 \rightarrow M_2 \rightarrow M_3$.

Without any loss of generality, we can assume that the jobs are processed according to sequence S, where S = 1, 2, …, n.

For sequence S, total rental cost of machines

$$R(S) = \sum_{j=1}^{3}\sum_{i=1}^{n}\left(p_{ij}+I_{ij}\right)x\,C_j$$

$$= \sum_{i=1}^{n}\begin{bmatrix}\left(p_{i1}+I_{i1}\right)x\,C_1+\left(p_{i2}+I_{i2}\right)x\,C_2+\\ \left(p_{i3}+I_{i3}\right)x\,C_3\end{bmatrix}$$

$$= \sum_{i=1}^{n} p_{i1}\,x\,C_1+\sum_{i=1}^{n} I_{i1}\,x\,C_1+\sum_{i=1}^{n} p_{i2}\,x\,C_2 +\sum_{i=1}^{n} I_{i2}\,x\,C_2+\sum_{i=1}^{n} p_{i3}\,x\,C_3+\sum_{i=1}^{n} I_{i3}\,x\,C_3$$

Under ***Policy III***, $I_{i1} = 0\ \forall$ i and $I_{12} = I_{13} = 0$

Therefore, total rental cost of machines

$$R(S) = \sum_{i=1}^{n} p_{i1}\times C_1+\sum_{i=1}^{n} p_{i2}\times C_2+\sum_{i=1}^{n} p_{i3}\times C_3 +\sum_{i=2}^{n} I_{i2}\times C_2+\sum_{i=2}^{n} I_{i3}\times C_3 \qquad (1)$$

Here processing times p_{ij} and rental costs C_j are constants $\forall$ i and j.

Therefore, total rental cost of machines is minimum when $\sum_{i=2}^{n} I_{i2}\times C_2+\sum_{i=2}^{n} I_{i3}\times C_3$ is minimum.

i.e.,

$$\sum_{i=1}^{n} p_{i2}\times C_2+\sum_{i=1}^{n} p_{i3}\times C_3+\sum_{i=2}^{n} I_{i2}\times C_2+\sum_{i=2}^{n} I_{i3}\times C_3$$

is minimum.

B. Evaluation of Lower Bounds

The lower bound for any partial schedule is the *VALUE* obtained such that whatever be the order of the remaining jobs to follow that schedule the total rental cost of machines should never be less than the *VALUE*. The lower bound for any partial schedule J_r is obtained under the assumption that jobs of J_r' does not wait for processing on particular machine and jobs after completing the processing on this machine are not waiting for processing on the remaining machine as if the machines are always available for processing. This reduces the problem of a single machine processing and the criterion for obtaining the optimal order of single machine can be obtained.

III. ALGORTHM

Algorithm 1:

The Branch-and-Bound technique is applied to minimize the total rental cost of machines. The lower bound $LB[J_r]$ for any schedule J_r is evaluated through the following steps :

Step 1: Compute

(i) $g_1 = \sum_{i=1}^{n} p_{i1} + \min_{i\in J_r'} p_{i2}$

(ii) $g_2 = t(J_r, 2) + \sum_{i\in J_r'} p_{i2}$

(iii) $g = \max\,[g_1, g_2] - Z_{i_1,1}$; where i_1 is first job of schedule J_r .

Step 2: Compute

(i) $G_1 = \sum_{i=1}^{n} p_{i1} + \min_{i\in J_r'}\left[p_{i2}+p_{i3}\right]$

(ii) $G_2 = t(J_r, 2) + \sum_{i\in J_r'} p_{i2} + \min_{i\in J_r'} p_{i3}$

(iii) $G_3 = t(J_r, 3) + \sum_{i\in J_r'} p_{i3}$

(iv) $G = \max\,[G_1, G_2, G_3) - Z_{i_1,2}$

Step 3: Compute

$LB[J_r] = g \times C_2 + G \times C_3$

IV. EXAMPLE

Example 4.1: Consider 4-job, 3-machine sequencing problem with processing times as given in Table 1. The rental cost per unit time for M_2 and M_3 is 6 units and 2 units respectively. i.e., $C_2 = 6$ and $C_3 = 2$.

Table 1: Processing Times of Jobs on Machines

Jobs	Machines		
	M_1	M_2	M_3
1	2	5	7
2	6	6	8
3	3	9	2
4	7	4	2

Applying Algorithm 1;

For $J_r = (1)$;

Step 1:

(i) $g_1 = \sum_{i=1}^{n} p_{ij} + \min_{i \in J_r'} p_{i2}$

$= 18 + \min [6, 9, 4]$

$= 18 + 4 = 22$

(ii) $g_2 = t(J_r, 2) + \sum_{i \in J_r'} p_{i2}$

$= 7 + (6+9+4) = 26$

(iii) $g = \max [g_1, g_2] - Z_{i_1, 1}$

$= \max [22, 26] - 2$

$= 26 - 2 = 24$

Step 2:

(i) $G_1 = \sum_{i=1}^{n} p_{i1} + \min_{i \in J_r'} \left[p_{i2} + p_{i3} \right]$

$= 18 + \min [14, 11, 6$

$= 18 + 6 = 24$

(ii) $G_2 = t(J_r, 2) + \sum_{i \in J_r'} p_{i2} + \min_{i \in J_r'} p_{i3}$

$= 7+19+ \min [8, 2, 2]$

$= 26+2 = 28$

(iii) $G_3 = t(J_r, 3) + \sum_{i \in J_r'} p_{i3}$

$= 14 + (8+2+2)$

$= 14+12 = 26$

(iv) $G_4 = \max [G_1, G_2, G_3) - Z_{i_1, 2}$

$= \max [24, 28, 26] - 7$

$= 28 - 7 = 21$

Step 3:

$LB[1] = g \times C_2 + G \times C_3$

$= 26 \times 6 + 21 \times 2$

$= 144+42 = 186$

Similarly the lower bounds for partial schedule J_r = (2), (3) and (4) are 184, 182 and 188 respectively.

Minimum value of lower bound is 182 for J_r = (3). Therefore, J_r = (3) is the branching node.

Continuing in this way, the Branch-and-Bound algorithm is applied for evaluations of relevant lower bounds and the scheduling tree is formed. Both are given as in Table 2 and Figure1.

Table 2: Lower Bounds

J_r	G	G	LB[J_r]
1	24	21	186
2	24	20	184
3	24	19	182
4	24	22	188
31	24	22	188
32	24	23	190
34	24	22	188
21	24	20	184
23	29	26	226
24	25	23	196
2134	—	—	184
2143	—	—	184

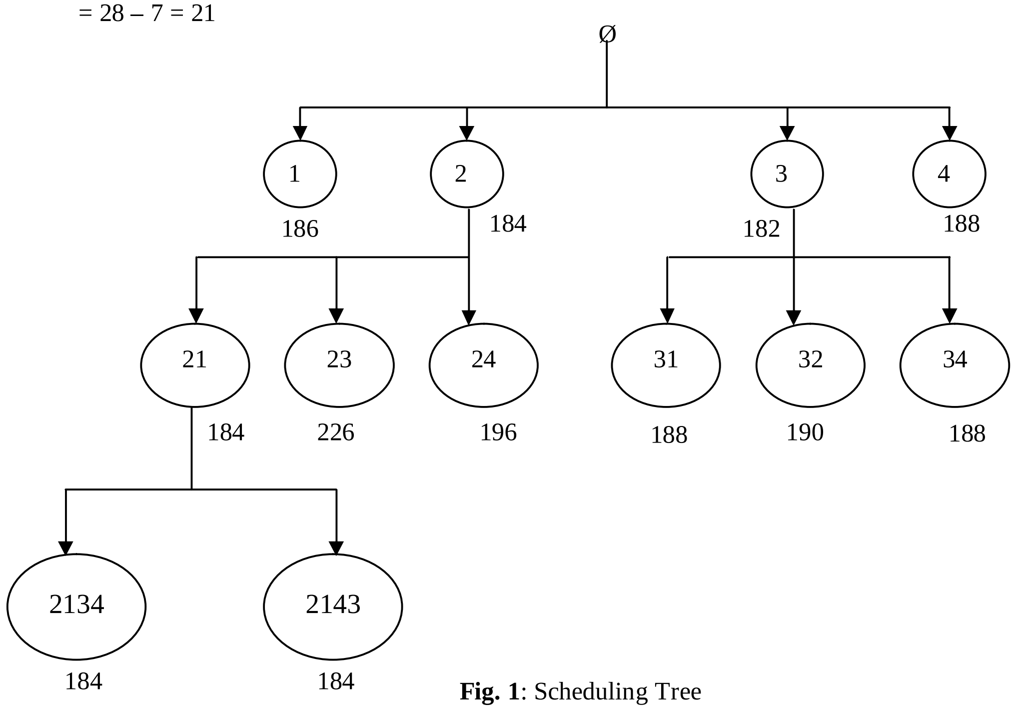

Fig. 1: Scheduling Tree

Hence, the optimal sequences are 2-1-3-4 and 2-1-4-3 with minimum rental cost of M_2 and M_3 as 184 units. Sequence 1-2-3-4 minimizes the total elapsed time. Therefore, it is not necessary that the sequences, which minimize the total elapsed time, will also minimize the total rental cost.

V. CONCLUSION

In this paper we have studied three-machine flow-shop problems under the situation when one has got the assignment but does not have one's own and has to take machines on rent to complete the assignment. Here, we have considered that first machine will be taken on rent in the starting of processing the jobs; second machine will be taken on rent when the first job is completed on first machine and third machine is taken on rent when first job is completed on second machine. We have developed a Branch-and-Bound algorithm which provides a sequence that minimizes the total rental cost of the machines.

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