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Symmetric Galerkin BEM for Non Linear Analysis of Historical Masonries

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Abstract-

The preservation of the historical and monumental buildings, but also of the considerable heritage of old constructions made by traditional techniques, is one of the actual problems of the structural mechanics. The level of knowledge of their structural behavior in presence of external actions is made through calculus methods and simple procedures in order to allow a reading of the material suffering degree and as a consequence of the related safety.

Unfortunately, often the masonry panels show openings located in an irregular way and cracks having small or big dimensions. In these cases the employment of strategies, as for example the identifying of the masonry piers and the transformation of the wall as a frame, prove to be inapplicable.

Recently, a new method was introduced, called Boundary Element Method, which applied in its symmetric formulation, in virtue of some peculiarities, proves to be better usable in comparison with other analysis methods.

In this paper an elastic analysis of walls, also in presence of geometrical nonlinearity consisting in the contact/detachment phenomenon among stone blocks. The wall having any shape and zone-wise variable physical characteristics is loaded in its plane. For these structures some interventions of structural strengthening have as aim to improve the wall behavior by reducing the stress concentration, so to have a better safety in comparison with its initial value.

Index Terms—**Contact/detachment, displacement approach, masonry, multidomain SGBEM.**

I. INTRODUCTION

Among the more considerable aspects within the protection of the masonry buildings, and in particular of the historical and monumental patrimony, there is the identifying of the static instability causes found in the walls. The difficulties in studying these structural systems depend on several elements, as the complexity of structural behavior and the uncertainties in the physical-mechanical characterization of the materials.

At same time, the choice of the interventions in order to guarantee the safety wanted, respectful of the rules of the restoration, is full of dangers. Often the architect or engineer uses empiric rules, also supported by rough structural schematizations, attaining to solutions often not answering to the necessary safety of the building.

As a consequence the use of the calculus code supported by appropriate computational methodologies is necessary to make numerical simulations which, once known the external actions, have the aim:

- to identify the possible causes of the instabilities through the solution of an inverse problem;
- to perform interventions on masonry panels, having feature of prevention towards possible static instabilities;
- to establish what, among the technical solutions of intervention to be activated, is this more appropriate, by making a comparison among different solutions of reinforcement.

The aim is to increase the safety conditions of the masonry buildings through the improving the stiffness of each wall panel in order to reach a global structural response with a more uniform field of the stress state.

In this paper the analysis is performed in the hypotheses that the actions (boundary and body forces, displacements of the constraint) act in the plane of each panel. It can be made through numerical simulations using analysis methodologies able to perform the necessary checks within reduced calculus times.

The usual calculus codes employed to analyze the masonries have as theoretical basis the Finite Element Method (FEM). But this method shows several drawbacks meanly connected to the discretization employed in the walls, usually foreseeing the use of elements having the same geometry.

The present paper suggests an innovative analysis based on the use of a multidomain strategy within the symmetric Galerkin formulation of the Boundary Element Method (SGBEM). Similar formulations were developed by several researches (Layton et al., 1997 [1]; Gray and Paulino, 1998 [2]; Vodicka et al., 2007 [3], Perez-Gavilan and Aliabadi [4]), all showing the mean peculiarities. Among these, the compatibility and the equilibrium conditions in each point of the domain are guaranteed because of the use of the fundamental solutions.

Within the SGBEM, in this paper, a strategy which uses the displacement method, proposed by Panzeca et al. [5-8] and reproposed by Terravecchia [9] and Zito et al. [10], is utilized. The latter method shows symmetry of the algebraic operators and is characterized as follows:

a) the subdivision of each masonry panel by substructures having any shape and dimension and different physical properties;

b) the boundary distinction of each substructure into constrained, free and interface with other substructures;

c) the writing of a characteristic elasticity equation for

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each substructure connecting weighted tractions evaluated on the interface boundary to nodal displacements of the same interface and to the load vector;
d) the use of the equation system through the writing of the compatibility strong form at the interface nodes and through the related weak form involving weighted (or generalized) tractions at the interface boundary elements,
e) the computation in closed form of all the double integrals making up the equation system coefficients, having hypersingular, singular or regular kernels [11, 12],
f) permit the transformation of the domain integrals into boundary ones. The reader can refer to Panzeca et al. [13] and Zito et al. [14, 15] for a more detailed discussion of the computational aspects and the related implementation techniques.

The discretization of the panel into substructures can be made through its subdivision into macrozones characterized by a homogenization of the physical parameters of the stone-mortar system and single elements constituting the panel as stone blocks and mortar layers, possibly considered separately.

The simultaneous use of the two different levels of discretization, one more sparse in macrozones and another more dense, is characteristic of the potenziality of the method. The presence of substructures having big or small dimensions does not involve numerical instabilities because all the coefficients of the equation system were computed in closed form.

The analysis of the masonry is made by using the Karnak.sGbem code (Cucco et al., 2002 [16]). This program allows to evaluate the response of the structural system subjected to all the possible static actions, whether volumetric or surface loads, but also to volumetric and linear distortions and to displacements imposed in the constraints.

Besides, it is possible to make a nonlinear analysis of cohesive detachment, so reproducing the evolution of the probable disconnectedness between stones through a strategy developed by some authors [17-18].

II. SGBEM MULTIDOMAIL ANALISYS

In this Section a brief synthesis of the Multidomain approach utilized, via SGBEM, is shown. The formulation, called displacement approach, was developed by Panzeca et al. [5], and the reader can make reference to [5-10] and to the related references to have wider explanations regarding the formulation and the related advantages.

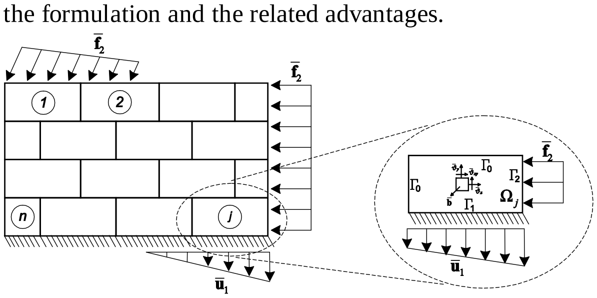

Fig.1. The masonry system subdivided into substructures.

Preliminary, a subdivision of the panel into substructures is made as in Fig.1 and, for each of these, a boundary distinction into constrained Γ_1, free Γ_2 and interface Γ_0 ones is made. For each boundary a discretization into boundary elements is performed to work in the discrete field in order to obtain an elasticity equation connecting quantities associated to the interface zones between contiguous substructures, that is

$$\mathbf{P}_{0i} = \mathbf{D}_{0ii}\ \mathbf{U}_{0i} + \hat{\mathbf{P}}_{0i} \quad \forall i = 1,...,n \tag{1}$$

n being the i-th substructure, obtained by the domain discretization.

In this equation:

$\mathbf{P}_{0i}$ represents the generalized traction vector of the i-th substructure defined at the interface elements of the boundary Γ_0 with contiguous ones and obtained as a response to the known and unknown actions. Each component is obtained through the weighting of the traction distribution on the boundary elements by using the shape function modelling the quantities, dual in energetic sense;

$\hat{\mathbf{P}}_{0i}$ characterizes the generalized traction vector of the *i-th* substructure obtained as a response to the known actions, regarding boundary and volume quantities;

$\mathbf{U}_{0i}$ is the displacement vector related all the interface nodes of the i-th substructure with contiguous ones;

$\mathbf{D}_{0ii}$ is the stiffness matrix of the i-th substructure, associated to the interface quantities.

A generalized Dirichlet condition on the constrained boundary regarding the weighted displacements;

a generalized Neumann condition on the free boundary regarding the weighted forces;

generalized Dirichlet and Neumann conditions on the interface boundary regarding the weighted displacements and tractions.

Through a condensation process shown in [5], a elasticity relation, called characteristic equation of the substructure, was obtained. This equation contains informations regarding the geometry of the substructure, the its physical-mechanical characteristics, and arises by using the generalized conditions defined on the boundary.

Subsequently, some appropriate regularity equations among contiguous substructures lead to a equation system by which it is possible to obtain the solution of the discretized system.

Indeed, eq.(1) is formally identical to that written within the FEM, but in this case with quantities referred to the interface nodes.

Let us consider for each substructure eq.(1). We can obtain a global relation connecting all the generalized traction vector with the related interface node displacement vector, i.e.:

$$\begin{vmatrix} \mathbf{P}_{01} \\ \vdots \\ \mathbf{P}_{0n} \end{vmatrix} = \begin{vmatrix} \mathbf{D}_{011} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{D}_{0nn} \end{vmatrix} \begin{vmatrix} \mathbf{U}_{01} \\ \vdots \\ \mathbf{U}_{0n} \end{vmatrix} + \begin{vmatrix} \hat{\mathbf{P}}_{01} \\ \vdots \\ \hat{\mathbf{P}}_{0n} \end{vmatrix} \tag{2}$$

formally equal to the eq. (1), i.e.

$$\mathbf{P}_0 = \mathbf{D}_0\ \mathbf{U}_0 + \hat{\mathbf{P}}_0 \tag{3}$$

Let us perform an identification (compatibility condition) among the displacement vector $\mathbf{U}_0$ collecting the displacements of each substructure and the displacement vector $\boldsymbol{\xi}$ of the assembled system.

$$\mathbf{U}_0 = \mathbf{H}_0\,\boldsymbol{\xi}_0 \tag{4}$$

where $\mathbf{H}$ is the topological matrix

$$\mathbf{H}_0 = \begin{vmatrix} \mathbf{H}_{01} \\ \vdots \\ \mathbf{H}_{0n} \end{vmatrix} \tag{5}$$

Let us introduce the equilibrium condition among generalized tractions at the interface boundaries.

$$\mathbf{H}_0^T\,\mathbf{P}_0 = \mathbf{0} \tag{6}$$

One obtains:

$$\mathbf{K}_0\,\boldsymbol{\xi}_0 + \hat{\mathbf{f}}_0 = \mathbf{0} \tag{7}$$

where the following positions are assumed:

$$\mathbf{K}_0 = \mathbf{H}_0^T\mathbf{D}_0\,\mathbf{H}_0 \quad \textit{and} \quad \hat{\mathbf{f}}_0 = \mathbf{H}_0^T\hat{\mathbf{P}}_0 \tag{8a,b}$$

$\mathbf{K}_0$ and $\hat{\mathbf{f}}_0$ being the stiffness matrix and the nodal force vector of the assembled system, involving interface elements among substructures, only.

The remaining nodal quantities, regarding the reactive forces of the constrained boundary, the displacements of the free boundary and the nodal mutual forces of the interface boundary among contiguous substructures are obtainable in the return back analysis.

III. CONTACT-DETACHMENT ITERATIVE PROBLEM AS LINEAR COMPLEMENTARITY PROBLEM (LCP)

Let the homogeneous elastic two-dimensional solid bodies A and B both be subjected to imposed displacements $\bar{\mathbf{u}}_1$ on Γ_1, to boundary forces $\bar{\mathbf{f}}_2$ on Γ_2 and to body forces $\bar{\mathbf{b}}$ in Ω. We suppose that friction does not occur between the two bodies and that simple contact or detachment is possible, only.

Let us introduce the coefficient $c \geq 0$ characterizing the cohesion between the boundaries in contact Γ_0^A and Γ_0^B, in the zone of potential detachment, and the distance vector modulus $|\boldsymbol{h}| \geq 0$ representing the distances between the corresponding points (reference gap) on the boundary nodes of Γ_2^A and Γ_2^B, in the zone of potential contact.

A. Continuum approach

Let us denote

- by $\mathbf{t}_0^A = -\mathbf{t}_0^B$ the stress vector acting between the contact points on the contact boundaries Γ_0^A and Γ_0^B respectively, $\mathbf{t}_2^A = \mathbf{t}_2^B = \mathbf{0}$ being verified on the free boundaries Γ_2^A and Γ_2^B, and
- by $\mathbf{u}_2^A \neq \mathbf{u}_2^B$ the displacement vectors at the detached boundaries Γ_2^A and Γ_2^B, $\mathbf{u}_0^A = \mathbf{u}_0^B$ being equal quantities between the displacements of the contact points on the contact boundaries Γ_0^A and Γ_0^B.

The boundary conditions of the contact-detachment problem are the following:

$$\mathbf{n}^A\left(\left(\mathbf{u}_2^A - \mathbf{u}_2^B\right) - \boldsymbol{h}\right) \leq 0\,, \quad c = 0 \quad \textit{gap condition}$$

$$\mathbf{n}^A\,\mathbf{t}_0^A - c \leq 0\,, \quad \boldsymbol{h} = \mathbf{0} \quad \textit{contact condition}$$

$$\left[\mathbf{n}^A\left(\left(\mathbf{u}_2^A - \mathbf{u}_2^B\right) - \boldsymbol{h}\right)\right]\left[\mathbf{n}^A\,\mathbf{t}_0^A - c\right] = 0 \quad \textit{compl. condition}$$

(9a-e)

valid at every point on the boundary subjected to the contact-detachment process, where $\mathbf{n}^A$ is the transpose of the normal vector associated with the boundary $\Gamma_0^A \cup \Gamma_2^A$ of the body A.

We describe the detachment process:

In the boundary zone marked by Γ_0, where the contact between the two bodies occurs ($c > 0$, $\boldsymbol{h} = \mathbf{0}$), the following conditions must be verified: $\mathbf{n}^A\left(\mathbf{u}_0^A - \mathbf{u}_0^B\right) = 0$ and $\mathbf{n}^A\,\mathbf{t}_0^A \leq c$. The detachment phenomenon is checked through the value assumed by the traction $\mathbf{t}_0^A$. In the case that the inequality (9c) is not satisfied, the point in contact is divided into two points belonging to the bodies *A* and *B* and, as a consequence, it causes the birth of different displacements on Γ_2^A and Γ_2^B.

We describe the contact process:

Vice versa, in the boundary zone marked by Γ_2, where contact between the two bodies ($c = 0, |\boldsymbol{h}| > 0$) does not exist, the following conditions have to be verified: $\mathbf{n}^A\left(\mathbf{u}_2^A - \mathbf{u}_2^B\right) \leq \mathbf{n}^A\boldsymbol{h}$ and $\mathbf{n}^A\,\mathbf{t}_0^A = 0$. The contact phenomenon is checked through the values assumed by the displacements $\mathbf{u}_2^A$ and $\mathbf{u}_2^B$ of the boundaries Γ_2^A and Γ_2^B of both the bodies. In the case that the inequality (9a) is not satisfied, the corresponding points join together and, as a consequence, it cause the birth of the traction between the two bodies in contact.

B. Discrete approach

Inside the topic of the SGBEM, to reach the analytical solution to this frictionless contact-detachment problem, an iterative Linear Complementarity Problem (LCP) can be employed once the elastic analysis has been performed by using Eq.(7).

Let us introduce a modelling of the displacements on the Γ_2^A and Γ_2^B boundary elements and of the tractions on the Γ_0^A and on the Γ_0^B, that are:

$$\mathbf{u}_2^A = \boldsymbol{\Psi}_{\mathrm{u}}\,\mathbf{U}_2^A\,, \quad \mathbf{u}_2^B = \boldsymbol{\Psi}_{\mathrm{u}}\,\mathbf{U}_2^B\,,$$
$$\mathbf{t}_0^A = \boldsymbol{\Psi}_{\mathrm{f}}\,\mathbf{T}_0^A, \quad \mathbf{t}_0^B = \boldsymbol{\Psi}_{\mathrm{f}}\,\mathbf{T}_0^B \tag{10a-d}$$

whose shape functions of the displacements and of the tractions are assumed equal for the bodies A and B, respectively.

$\mathbf{U}_2^A$ and $\mathbf{U}_2^B$ vectors collect the displacements of the nodes of the free boundary Γ_2^A and Γ_2^B, whereas $\mathbf{T}_0^A$ and $\mathbf{T}_0^B$ collect the tractions evaluated at the extremes of the boundary elements of the contact boundary Γ_0^A and Γ_0^B.

As a consequence of the boundary discretization into boundary elements, the symmetric approach foresees the weighting of all the terms present in the system (9). In particular, eq.(9a), which imposes a constraint on the displacements, has to be weighted using the shape functions modeling the tractions, whereas eq.(9b), which introduce a constraint on the tractions, has to be weighted using the shape functions of the displacements.

One has:

$$\mathbf{N}_2^A\left(\left(\int_{\Gamma_2^A}\boldsymbol{\psi}_f\mathbf{u}_2^A\,d\Gamma_2^A-\int_{\Gamma_2^B}\boldsymbol{\psi}_f\mathbf{u}_2^B\,d\Gamma_2^B\right)-\int_{\Gamma_2^A}\boldsymbol{\psi}_f\boldsymbol{h}\,d\Gamma_2^A\right)\le\mathbf{0}\,,$$

$$\int_{\Gamma_0^A}\boldsymbol{\psi}_u c\,d\Gamma_0^A=\mathbf{0}\qquad \textit{gap condition}$$

(11a,b)

$$\mathbf{N}_0^A\int_{\Gamma_0^A}\boldsymbol{\psi}_u\mathbf{t}_0^A\,d\Gamma_0^A-\int_{\Gamma_0^A}\boldsymbol{\psi}_u c\,d\Gamma_0^A\le\mathbf{0}\,,$$

$$\int_{\Gamma_2^A}\boldsymbol{\psi}_f\boldsymbol{h}\,d\Gamma_2^A=\mathbf{0}\qquad \textit{contact condition}\qquad\text{(11c,d)}$$

$$\left[\mathbf{N}_2^A\left(\left(\int_{\Gamma_2^A}\boldsymbol{\psi}_f\mathbf{u}_2^A\,d\Gamma_2^A-\int_{\Gamma_2^B}\boldsymbol{\psi}_f\mathbf{u}_2^B\,d\Gamma_2^B\right)-\int_{\Gamma_2^A}\boldsymbol{\psi}_f\boldsymbol{h}\,d\Gamma_2^A\right)\right]$$

$$\left[\mathbf{N}_0^A\int_{\Gamma_0^A}\boldsymbol{\psi}_u\mathbf{t}_0^A\,d\Gamma_0^A-\int_{\Gamma_0^A}c\,d\Gamma_0^A\right]=0\;\; \textit{compl. condition}$$

(11e)

where $\mathbf{N}_2^A=diag\left(\cdots\;\mathbf{n}_i^A\;\cdots\right)$ and $\mathbf{N}_0^A=diag\left(\cdots\;\mathbf{n}_j^A\;\cdots\right)$ with *i*, *j* defining the boundary elements on Γ_2^A and Γ_0^A, respectively, involved in the contact-detachment process.

In compact form the previous system (11a-e) can be written in the following way:

$$\mathbf{N}_2^A\left(\left(\mathbf{W}_2^A-\mathbf{W}_2^B\right)-\mathbf{H}\right)\le\mathbf{0}\,,\quad \mathbf{C}=\mathbf{0}\qquad \textit{gap condition}$$

$$\mathbf{N}_0^A\;\mathbf{P}_0^A-\mathbf{C}\le\mathbf{0}\,,\quad \mathbf{H}=\mathbf{0}\qquad \textit{contact condition}$$

$$\left[\mathbf{N}_2^A\left(\left(\mathbf{W}_2^A-\mathbf{W}_2^B\right)-\mathbf{H}\right)\right]\left[\mathbf{N}_0^A\;\mathbf{P}_0^A-\mathbf{C}\right]=0\;\textit{compl. condition}$$

(12a-e)

where the following positions have been introduced:

$$\mathbf{W}_2^A=\int_{\Gamma_2^A}\boldsymbol{\psi}_f\mathbf{u}_2^A\,d\Gamma_2^A$$

$$\mathbf{W}_2^B=\int_{\Gamma_2^B}\boldsymbol{\psi}_f\mathbf{u}_2^B\,d\Gamma_2^B$$

$$\mathbf{P}_0^A=\int_{\Gamma_0^A}\boldsymbol{\psi}_u\mathbf{t}_0^A\,d\Gamma_0^A$$

(13a-c)

and where the vector $\mathbf{H}$ collects all the nodal gaps between the corresponding nodes of the boundaries Γ_2^A and Γ_2^B, in the zone of potential contact, whereas the vector $\mathbf{C}$ collects the cohesion between the nodes which are in contact, in the zone of potential detachment Γ_0, defined as follows:

$$\mathbf{H}=\int_{\Gamma_2^A}\boldsymbol{\psi}_f\boldsymbol{h}\,d\Gamma_2^A,\qquad \mathbf{C}=\int_{\Gamma_0^A}\boldsymbol{\psi}_u c\,d\Gamma_0^A\qquad\text{(13d,e)}$$

The displacements $\mathbf{u}_2^A$ and $\mathbf{u}_2^B$, and so also the traction $\mathbf{t}_0^A$, present in the previous LCP system (10a-c), are obtained by using the Somigliana Identities as follows

$$\mathbf{u}_2^A=\int_{\Gamma_1^A}\mathbf{G}_{uu}\boldsymbol{\psi}_f\mathbf{F}_1^A+{-\!\!\!\!\!\!\int_{\Gamma_2^A}}\mathbf{G}_{ut}\boldsymbol{\psi}_u(-\mathbf{U}_2^A)+\frac{1}{2}\boldsymbol{\psi}_u(-\mathbf{U}_2^A)+$$
$$\int_{\Gamma_0^A}\mathbf{G}_{uu}\boldsymbol{\psi}_f\mathbf{T}_0^A+\int_{\Gamma_0^A}\mathbf{G}_{ut}\boldsymbol{\psi}_u(-\mathbf{U}_0^A)+\hat{\mathbf{u}}_2^A$$

(14a)

$$\mathbf{u}_2^B=\int_{\Gamma_1^B}\mathbf{G}_{uu}\boldsymbol{\psi}_f\mathbf{F}_1^B+{-\!\!\!\!\!\!\int_{\Gamma_2^B}}\mathbf{G}_{ut}\boldsymbol{\psi}_u(-\mathbf{U}_2^B)+\frac{1}{2}\boldsymbol{\psi}_u(-\mathbf{U}_2^B)+$$
$$\int_{\Gamma_0^B}\mathbf{G}_{uu}\boldsymbol{\psi}_f\mathbf{T}_0^B+\int_{\Gamma_0^B}\mathbf{G}_{ut}\boldsymbol{\psi}_u(-\mathbf{U}_0^B)+\hat{\mathbf{u}}_2^B$$

(14b)

$$\mathbf{t}_0^A=\int_{\Gamma_1^A}\mathbf{G}_{tu}\boldsymbol{\psi}_f\mathbf{F}_1^A+\int_{\Gamma_2^A}\mathbf{G}_{tt}\boldsymbol{\psi}_u(-\mathbf{U}_2^A)+{-\!\!\!\!\!\!\int_{\Gamma_0^A}}\mathbf{G}_{tu}\boldsymbol{\psi}_f\mathbf{T}_0^A-$$
$$\frac{1}{2}\boldsymbol{\psi}_f\mathbf{T}_0^A+\int_{\Gamma_0^A}\mathbf{G}_{tt}\boldsymbol{\psi}_u(-\mathbf{U}_0^A)+\hat{\mathbf{t}}_0^A$$

(14c)

where, in addition to the previous definition, $\mathbf{F}_1^A$ and $\mathbf{F}_1^B$ vectors are the nodal reaction vectors of the constrained boundaries Γ_1^A and Γ_1^B and $\hat{\mathbf{u}}_2^A,\hat{\mathbf{u}}_2^B,\hat{\mathbf{t}}_0^A$ vectors are the response to the known actions, as body forces and volumetric distortions in Ω domain, imposed displacements on the constrained boundary.

IV. Examples

To show the effectiveness of the analysis method via SGBEM, two examples of masonry structures, both subjected to body actions and to horizontal or vertical distribution of forces, are shown, through a loading process and analyzing the effects produced.

A. Masonry panel

Let be consider a masonry panel made by stone blocks (see Fig.2), placed by a thin layer of setting mortar and surmounted by a stringcourse in concrete. The panel is subjected to the body forces and to two distributed loads, both acting along the stringcourse, vertical constant $q_y=10000daN/m$ and horizontal variable force q_x.

The cohesion among stone blocks and the mortar is assumed $c=0.4MPa$, whereas a boundary discretization of all the substructures (blocks and stringcourse) is made with step of $d=10cm$.

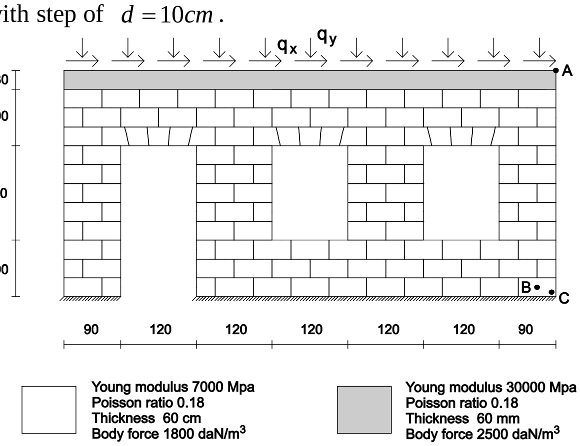

Fig.2. The masonry panel subdivided into substructures.

The hypotheses on whose the non linear analysis was performed are:

- presence of infinite friction,
- absence of fracture which could to start in some blocks otherwise to the detachment among blocks,

- absence of the plastic phenomenon, located in some blocks or diffuse,
- the panel is perfectly constrained at the soil.

The recursive analysis has been made by supposing that, during the load process, the resulting horizontal force $Q_x = q_x\, l$ assumes a unitary increment (1 *Ton*), where l is the length of the panel.

In Fig.3 the detachments are shown through marked lines, happened when the horizontal resultant load takes on the values $Q_x = 10, 20, 30, 40, 50, 56\ Tons$, whereas in Fig.4 the original and final shape configurations are shown.

In Fig.5 the function $Q_x = f(u_x)$, which relates the horizontal load with the horizontal displacement of the point A, is shown.

When the force takes on the value near to $Q_x = 56\ Tons$, the curve shows a almost constant course, so characterizing the beginning of the collapse phenomenon. In effect, if one proceeds by the analysis with load increment, the curve begins again to increase with a weak slope, so showing the hardening phenomenon, and it happens because of the hypotheses introduced.

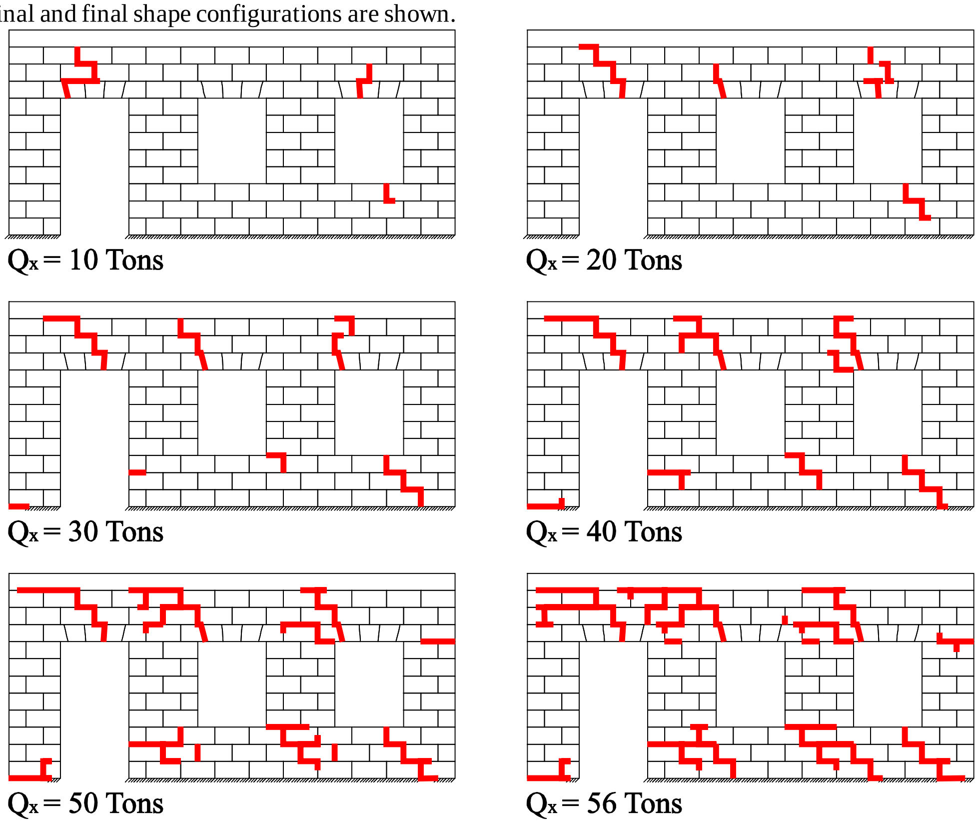

Fig.3. The detachment phenomenon in the different load conditions.

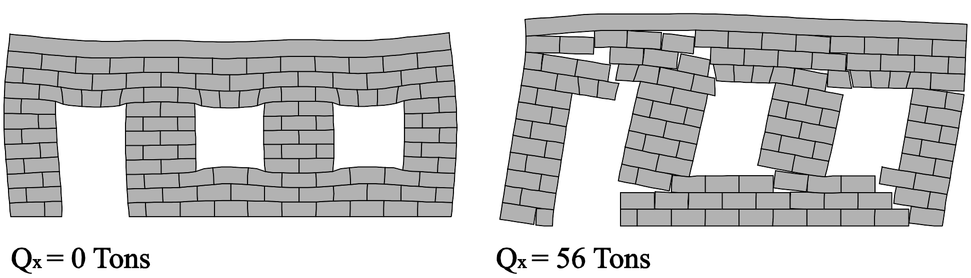

Fig.4. The masonry panel in the original and final shape configurations.

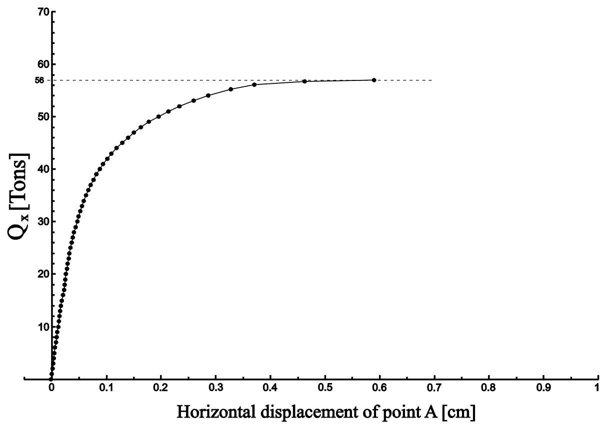

Fig.5. The load - displacement curve.

B. Circular vault

In this second example we consider a depressed vault with abutments made by two little vaults, both in ashlars of stones. The geometrical and physical characteristics are shown in Fig.6. Among all stones the presence of mortar joints was simulated through the introduction of an interface subjected to damage with value of cohesion $c = 0.4$ Mpa .

The aim of this application is to show how with the use of this formulation it is possible to understand the geometrically non-linear phenomena and also to show how allows to quantify the intervention proposal for improvement thanks to the use of the formulation extremely advantageous as the SGBEM for the reasons that have already been discussed above.

For this application we have implemented two different strategies, the first is the update of the geometry, the second one keeps the geometry unchanged. The update of the geometry is a strategy that consists in modifying, through an incremental process, the kinematics of the structural system. This is simplified by the absence of distinction between elastic and inelastic quantities, then in a step by step process the displacement of any point of the structure becomes a change of geometry. The example given provides for the vault initially subjected to mass forces alone (constant in time) and to a variable load acting on the extrados of the ceiling that from $q = 0$ reaches the final value $q = 800$ da / m . The presence of these actions leads to the birth of a crack pattern that will be discussed in detail. Subsequently, a metal chain, subject to a thermal load that from a null value reaches a final value $-\Delta T = 15\,°C$, is introduced.

The presence of the negative thermal action is introduced to simulate the process of incremental stress of the metal rod through metal heads and threaded bolts. The incremental process relates in a first phase the increase of the load q. In this phase, the presence of the variable load and the mass forces (constant) lead to the arise of a crack pattern that is shown in Fig.8a. In it the evolution of the process, obtained through the updating of the geometry and in the absence of update, is compared.

In a next step we introduce the metal rod and increase the thermal load $-\Delta T$ in order to restore the physical continuity among stones by closing the cracks caused by gravity loads. This effect is clearly shown in Fig.7b. In it the improvement effect caused by the introduction of the tie rod subjected to a negative temperature change, that converts the distorting effect, is shown.

The analysis of Fig.7 requires further comments to highlight both the peculiarities of the proposed method and the differences of the analyses (updating and not updating of the geometry) during the load increment q. The two approaches show approximately the same crack pattern and the gaps are concentrated at the key and support areas and also at the pier supports. In particular, the arch is more pliable when analyzing the system without updating the geometry, stiffer if we analyze the system with updated of the geometry.

The two approaches show a substantial difference in the initial phase of introduction of the tie rod, that is when $-\Delta T = 0$. Indeed, in the simulation obtained by updating of the geometry the introduction of the tie rod does not alter the system and the crack pattern is identical if compared to that obtained in the condition $q = 800$ daN / m ; vice versa in the approach without updating of the geometry the only presence of the tie rod, in absence of thermal load, causes a significant improvement characterized by an instantaneous closing nearly of all areas of detachment. The occurrence of this phenomenon goes away from the reality because only the presence of a pre-stress in the tie rod can produce benefits to the system.

It is noted that when the final value of the thermal load $-\Delta T = 15\,°C$ is reached, the two approaches give results appreciably similar in terms of the presence of detachments.

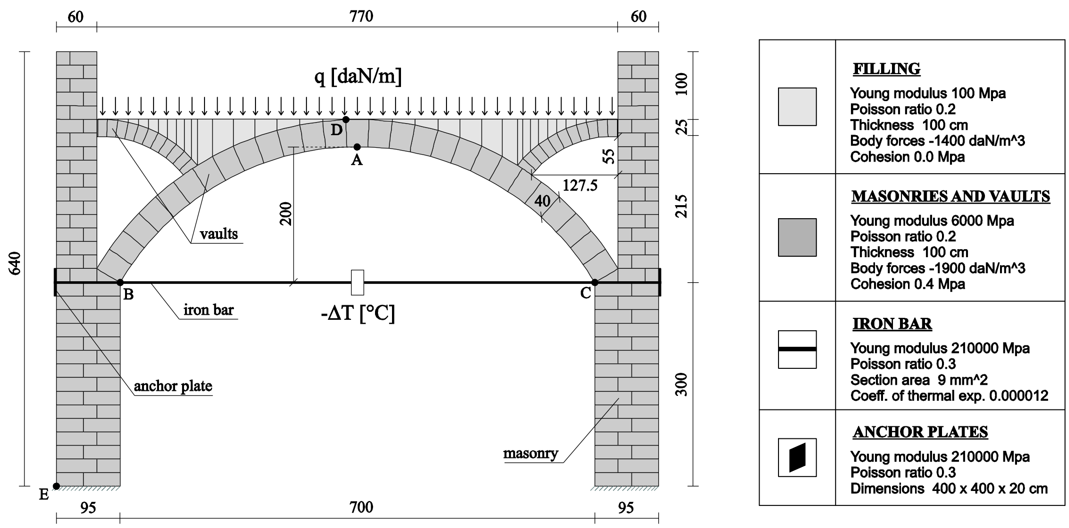

Fig.6. The vault subdivided into substructures.

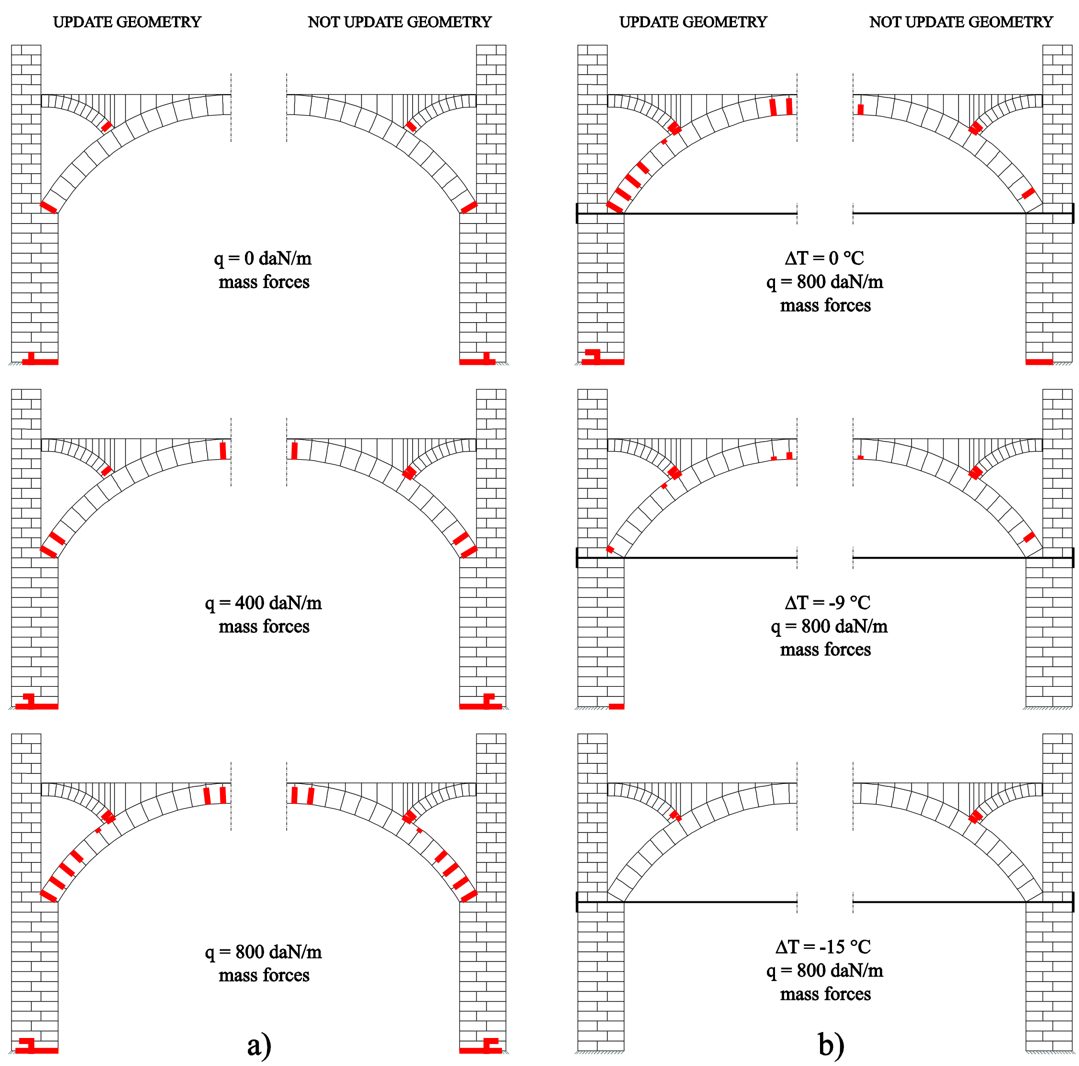

Fig.7. The detachment phenomenon in the different load conditions:
a) distributed load q, b) thermal load $-\Delta T$.

V. Conclusions

The preservation and the safety degree definition of the historical and monumental buildings has become a topic of big interest only recently, it involving a lot of practical engineering fields as that of chemist, of materials and of structures. Particularly, the engineering of the structure assumed a central role because the preservation of monumental buildings cannot in any case leave out the safety degree as a response to all the foreseen and foreseeable actions.

At this aim, the present paper shows an innovative methodology of SGBEM multidomain type for the structural analysis of masonry systems by utilizing the advantages of the SGBEM use either during the phase of assessment of the instability cause or in the choice of the more idoneous statical preservation technique.

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