

A Survey of Deep Learning Techniques for Time-Series Forecasting

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ABSTRACT- Various deep learning architectures have been created to cater to the diverse nature of time-series datasets across various domains. This article examines prevalent designs for both encoder and decoder components used in time-series forecasting, covering both one-step-ahead and multi-horizon approaches. It delves into the incorporation of temporal information into predictions by each model. Additionally, recent advancements in hybrid deep learning models are highlighted, showcasing the integration of well-established statistical models with neural network elements to enhance methods in both categories. Lastly, the article outlines how deep learning can contribute to decision support in the context of time-series data. This piece is part of the theme issue on 'Machine learning for weather and climate modeling.

KEYWORDS- Time-Series Modeling, Deep Learning, Neural Networks, Forecasting Models, Machine Learning.

I. INTRODUCTION

Time-series modeling has been a prominent subject of academic exploration, playing a crucial role in diverse fields like climate modeling [1], biological sciences [2], medicine [3], and practical domains like retail [4] and finance [5]. Traditional approaches have predominantly relied on parametric models guided by domain expertise, such as autoregressive (AR) [6], exponential smoothing [7,8], or structural time-series models [9]. However, contemporary machine learning techniques offer a data-centric approach to capturing temporal dynamics [10]. With the surge in data accessibility and computing capabilities, machine learning has emerged as an indispensable component of the next wave of time-series forecasting models.

In recent times, deep learning has become widely recognized, driven by impressive accomplishments in tasks such as image classification [11], natural language processing [12], and reinforcement learning [13]. Deep neural networks leverage specialized architectural assumptions, known as inductive biases [14], that capture the intricacies of the underlying datasets. This enables these networks to acquire intricate data representations [15], reducing the reliance on manual feature engineering and model design. The accessibility of open-source Backpropagation frameworks [16,17] has further streamlined network training, facilitating customization of network components and loss functions.

Given the variety of time-series challenges found in different fields, numerous design options for neural networks have surfaced. This article provides an overview of prevalent strategies for time-series prediction using deep neural networks. Initially, we present cutting-edge techniques applicable to common forecasting issues, such as multi-horizon forecasting and uncertainty estimation. Subsequently, we examine the rise of a novel trend involving hybrid models, which integrate domain-specific quantitative models with deep learning elements to enhance forecasting accuracy. Following that, we delineate two fundamental approaches in which neural networks can contribute to decision support, focusing on interpretability and counterfactual prediction methods. Lastly, we summarize with insights into promising future research avenues in deep learning for time series prediction, particularly in the realms of continuous-time and hierarchical models.

II. LITERATURE REVIEW

A. Deep Learning Architectures For Time-Series Forecasting

Time-series forecasting models anticipate future values of a target variable, denoted as $y_{i,t}$ for a specified entity i at time t . Each entity signifies a coherent collection of temporal data, such as readings from various weather stations in climatology or vital signs from diverse patients in medicine. These entities can be observed simultaneously. In its most straightforward manifestation, one-step-ahead forecasting models adopt the following structure:

$$\hat{y}_{i,t+1} = f(y_{i,t-k:t}, x_{i,t-k:t}, s_i),$$

In this context, $\hat{y}_{i,t+1}$ represents the model's forecast, and $y_{i,t-k:t} = \{y_{i,t-k}, \dots, y_{i,t}\}$, $x_{i,t-k:t} = \{x_{i,t-k}, \dots, x_{i,t}\}$ denote the observations of the target and exogenous inputs, respectively, within a look-back window of size k . The variable s_i represents static metadata associated with the entity, such as sensor location, and $f(\cdot)$ represents the prediction function learned by the model. Although our primary focus in this survey is on univariate forecasting (i.e., 1-D targets), it's important to note that the same components can be extended to multivariate models without loss of generality [26–30]. For the sake of simplicity in notation, we will omit the entity index i in subsequent sections unless explicitly necessary.

B. Basic Building Blocks

Predictive relationships are acquired by deep neural networks through the utilization of nonlinear layers to create intermediate feature representations [15]. In the context of time-series scenarios, this process can be interpreted as encapsulating pertinent historical information within a latent variable, denoted as z_t . The ultimate forecast is then generated solely based on the value of z_t .

$$f(y_{t-k:t}, x_{t-k:t}, s) = g_{dec}(z_t),$$

And

$$z_t = g_{enc}(y_{t-k:t}, x_{t-k:t}, s)$$

The encoder and decoder functions, denoted as $g_{enc}(\cdot)$ and $g_{dec}(\cdot)$, respectively, serve as fundamental components in deep learning architectures. It's important to note that in equation (2.1), the subscript i has been omitted for simplicity in notation, such as replacing $y_{i,t}$ with y_t . These encoders and decoders are essential building blocks, and the selection of the network determines the types of relationships that the model can learn. This section delves into contemporary design choices for encoders, as depicted in Figure 1, and their connections to traditional temporal models. Additionally, we investigate common network outputs and loss functions employed in time-series forecasting applications.

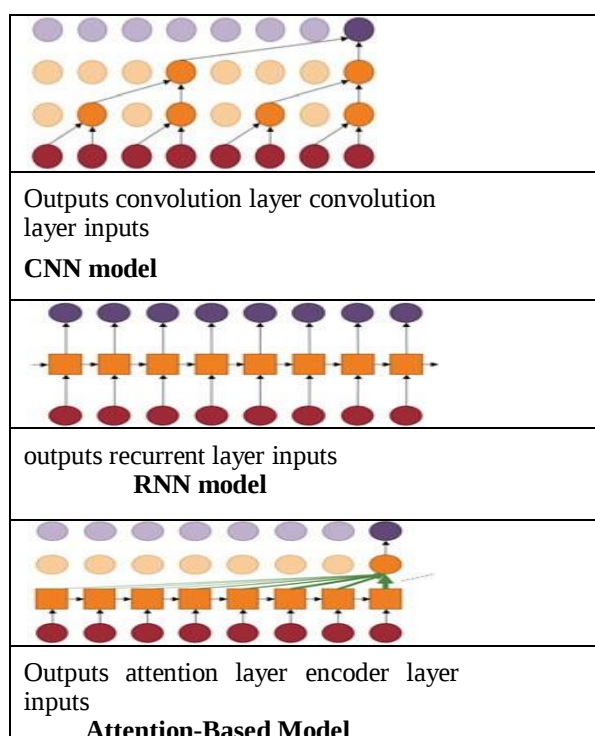


Figure 1: Incorporating temporal information using different encoder architectures. (a) CNN model, (b) RNN model and (c) attention-based model. (Online version in colour.)

C. Convolution Neural Networks

Originally designed for image datasets, convolution neural networks (CNNs) have been conventionally employed to capture local relationships that remain consistent across spatial dimensions [11,31]. To make CNNs suitable for time-series datasets, researchers incorporate multiple layers of causal convolutions [32–34]. These causal convolutions involve filters specifically designed to utilize only past information for forecasting. In the case of an intermediate feature at hidden layer l , each causal convolution filter follows the outlined form.

$$h_t^{l+1} = A((W * h)(l * t))$$

And,

$$(W * h)(l * t) = \sum_{\tau=0}^k W(l, \tau) h_{t-\tau}^l,$$

where $h^l \in R^{H_{in}}$ is an intermediate state at layer l at time t , $*$ is the convolution operator, $W(l, \tau) \in R^{H_{out} \times H_{in}}$ is a fixed filter weight at layer l , and $A(\cdot)$ is an activation function, such as a sigmoid function, representing any architecture-specific nonlinear processing. For CNNs that use a total of L convolution layers, we note that the encoder output is then $z_t = h^L$.

Examining the 1-D scenario, it becomes evident that equation (2.5) shares a striking resemblance with finite impulse response (FIR) filters in digital signal processing [35]. This observation gives rise to two significant implications concerning the temporal relationships learned by CNNs. Firstly, akin to the spatial invariance assumptions inherent in standard CNNs, temporal CNNs operate under the assumption that relationships are time-invariant. This entails utilizing the same set of filter weights across all time steps. Moreover, CNNs are constrained to utilize inputs within their designated look back window or receptive field for making predictions. Consequently, careful tuning of the receptive field size (k) is essential to ensure that the model effectively incorporates all pertinent historical information. It is noteworthy that a single causal CNN layer is essentially equivalent to an autoregressive (AR)

D. Recurrent Neural Networks

Historically, recurrent neural networks (RNNs) have found application in sequence modeling [31], demonstrating notable performance across various natural language processing tasks [36]. Due to the inherent representation of time-series data as sequences of inputs and targets, numerous RNN-based architectures have been created for temporal forecasting purposes [37–40]. Fundamentally, RNN cells incorporate an internal memory state that serves as a concise recapitulation of preceding information. This memory state undergoes recursive updates with new observations at each time step, as depicted in figure 1b.

$$z_t = v(z_{t-1}, y_t, x_t, s)$$

where $z_t \in R^H$ here is the hidden internal state of the RNN, and $v(\cdot)$ is the learnt memory update function. For

instance, the Elman RNN [41], one of the simplest RNN variants, would take the form below

$$t_{t+1} = \gamma_y (W_y z_t + b_y)$$

and

$$z_t = \gamma_z (W_{z1} z_{t-1} + W_{z2} y_t + W_{z3} x_t + W_{z4} s + b_z)$$

where W , b are the linear weights and biases of the network respectively, and $\gamma_y(\cdot)$, $\gamma_z(\cdot)$ are network activation functions. Note that RNNs do not require the explicit specification of a lookback window as per the CNN case. From a signal processing perspective, the main recurrent layer—i.e. equation (2.9)—thus resembles a nonlinear version of infinite impulse response (IIR) filters.

The challenge of learning long-range dependencies in data arises in older versions of Recurrent Neural Networks (RNNs) due to limitations in their ability to handle infinite lookback windows. This issue is attributed to problems with exploding and vanishing gradients, as discussed in previous studies [42,43] and related to resonance in the memory state [31]. To overcome these limitations, Long Short-Term Memory networks (LSTMs) [44] were developed. LSTMs enhance gradient flow within the network by introducing a cell state (ct) that stores long-term information. This information is modulated through a series of gates, addressing the challenges posed by older RNN variants.

$$\text{input gate: } i_t = \sigma(W_{i1} z_{t-1} + W_{i2} y_t + W_{i3} x_t + W_{i4} s)$$

$$\text{output gate: } o_t = \sigma(W_{o1} z_{t-1} + W_{o2} y_t + W_{o3} x_t + W_{o4} s)$$

$$\text{forget gate: } f_t = \sigma(W_{f1} z_{t-1} + W_{f2} y_t + W_{f3} x_t + W_{f4} s)$$

where z_{t-1} is the hidden state of the LSTM, and $\sigma(\cdot)$ is the sigmoid activation function. The gates modify the hidden and cell states of the LSTM as below

$$\text{hidden state: } z_t = O_t \odot \tanh(c_t)$$

$$\text{cell state: } c_t = f_t \odot c_{t-1} + i_t \odot \tanh(W_{c1} z_{t-1} + W_{c2} y_t + W_{c3} x_t + W_{c4} s + b_c)$$

where $\odot$ is the element-wise (Hadamard) product, and $\tanh(\cdot)$ is the tanh activation function.

In [39], the connection between Bayesian filters [45] and Recurrent Neural Networks (RNNs) is explored. Both Bayesian filters and RNNs share a common characteristic in maintaining a hidden state that undergoes recursive updates over time. In the case of Bayesian filters, exemplified by the Kalman filter [46], the inference process involves updating the sufficient statistics of the latent state through a sequence of state transition and error correction steps. The deterministic equations employed in Bayesian filtering steps to modify sufficient statistics can be likened to the RNN's simultaneous approximation of both steps. In this analogy, the memory vector in the RNN encompasses all pertinent information necessary for making predictions.

E. Attention Mechanisms

The advancement of attention mechanisms [47,48] has resulted in enhanced capabilities for learning long-term dependencies. Transformer architectures, in particular, have demonstrated state-of-the-art performance across various natural language processing applications [12,49,50]. Temporal features are aggregated by attention layers using dynamically generated weights (illustrated in figure 1c). This enables the network to concentrate on crucial time steps from the past, even when they are significantly distant within the lookback window. In essence, attention serves as a mechanism for key-value lookup based on a provided query [51], as outlined below.

$$h_t = \sum_{\tau=0}^k \alpha(k_t * q_\tau) v_{\tau-\tau},$$

where the key κ_t , query q_τ and value $v_{\tau-\tau}$ are intermediate features produced at different time steps by lower levels of the network. Furthermore, $\alpha(\kappa_t, q_\tau) \in [0, 1]$ is the attention weight for $t - \tau$ generated at time t , and h_t is the context vector output of the attention layer.

Note that multiple attention layers can also be used together as per the CNN case, with the output from the final layer forming the encoded latent variable z_t .

Recent research has highlighted the advantages of incorporating attention mechanisms into time-series forecasting applications, showcasing enhanced performance compared to similar recurrent networks [52–54]. In one example, attention is employed to consolidate features extracted by RNN encoders, generating attention weights as described below in [52].

$$\alpha(t) = \text{softmax}(\eta_t)$$

And

$$\eta_t = W_{\eta_1} \tanh(W_{\eta_2} k_{t-1} + W_{\eta_3} q_\tau + b_\eta)$$

where $\alpha(t) = [\alpha(t, 0), \dots, \alpha(t, k)]$ is a vector of attention weights, κ_{t-1} , q_t are outputs from LSTM encoders used for feature extraction, and $\text{softmax}(\cdot)$ is the softmax activation function.

In recent times, transformer architectures have been explored in [53,54], utilizing scalar-dot product self-attention [49] on features extracted within a specified look back window. From the viewpoint of time-series modeling, attention offers two crucial advantages. First and foremost, networks with attention can focus directly on noteworthy events. In applications such as retail forecasting, this capability allows the model to consider events like holidays or promotions, which may impact sales positively. Secondly, as demonstrated in [54], attention-based networks can acquire the ability to learn temporal dynamics specific to different regimes, achieving this by employing distinct attention weight patterns for each regime.

F. Multi-Horizon Forecasting Models

In numerous scenarios, having the ability to obtain predictive estimates at various future time points proves advantageous. This enables decision-makers to observe trends over an extended period, allowing for the optimization of actions across the entire forecasted trajectory. From a statistical standpoint, the concept of

multi-horizon forecasting can be perceived as a slight adjustment to the conventional one-step-ahead prediction problem, as illustrated in equation (2.1) below.

$$\hat{y}_{t+\tau} = f(y_{t-k:t}, x_{t:k:t}, u_{t-k:t+\tau}, s, \tau)$$

where $\tau \in \{1, \dots, \tau_{\max}\}$ is a discrete forecast horizon, u_t are known future inputs (e.g. date information, such as the day-of-week or month) across the entire horizon, and x_t are inputs that can only be observed historically.

In line with traditional econometric approaches [57,58], deep learning architectures for multi-horizon forecasting can be divided into iterative and direct methods—as shown in figure 2 and described in detail below.

G. Iterative Methods

Multi-horizon forecasting often involves iterative methods that leverage AR deep learning architectures [37,39,40,53]. These approaches generate forecasts for multiple time horizons by iteratively inputting target samples into future time steps, as illustrated in figure 2a. The process is repeated to create various trajectories, and forecasts are derived by utilizing sampling distributions for target values at each step. Predictive means, such as the Monte Carlo estimate, can be extracted through this methodology.

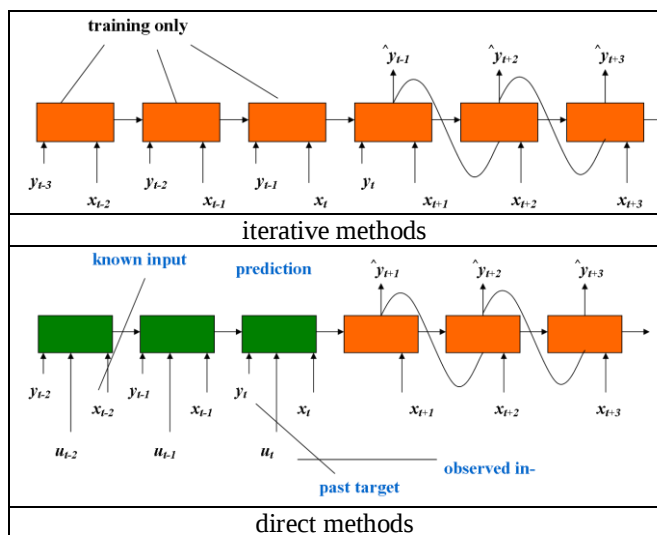


Figure 2: Main types of multi-horizon forecasting models. (a) Iterative methods (b) direct methods. (Online version in color.)

AR models are trained using the same method as one-step-ahead prediction models, employing backpropagation through time. The iterative approach facilitates the seamless extension of standard models to multi-step forecasting. However, since a minor amount of error is generated at each time step, the recursive nature of iterative methods may result in significant error accumulation over extended forecasting periods. Furthermore, these methods operate under the assumption that all inputs except the target are known at runtime, necessitating only target samples to be inputted into future time steps. This assumption can pose limitations in practical scenarios where observed inputs are present, highlighting the necessity for more adaptable methods.

H. Direct Methods

Direct methods overcome the drawbacks associated with iterative methods by generating forecasts directly through the utilization of all available inputs. Typically employing sequence-to-sequence architectures [52, 54, 56], these methods involve an encoder to succinctly summarize past information (including targets, observed inputs, and a priori known inputs) and a decoder to integrate this information with known future inputs, as illustrated in figure 2b. Another approach, detailed in [59], involves the use of simpler models to directly generate a fixed-length vector corresponding to the desired forecast horizon. However, this approach necessitates the specification of a maximum forecast horizon ($\tau_{\max}$), with predictions only made at predefined discrete intervals.

III. CONCLUSION AND FUTURE DIRECTIONS

With the recent increase in data accessibility and computational capabilities, deep neural network architectures have garnered considerable success in addressing forecasting challenges across various domains. This article explores the primary architectures employed in time-series forecasting, shedding light on the essential components integral to neural network design. The examination delves into how these architectures incorporate temporal information to make one-step-ahead predictions and discusses their adaptability for multi-horizon forecasting. Additionally, we outline the emerging trend of hybrid deep learning models, which synergize statistical and deep learning elements to surpass the performance of purely specialized methods. Lastly, we provide an overview of two approaches to extending deep learning for enhanced decision support over time, with a focus on methods related to interpretability and counterfactual prediction.

Despite the numerous deep learning models designed for time-series forecasting, certain limitations persist. Firstly, deep neural networks typically necessitate time series to be discretized at regular intervals, posing challenges in forecasting datasets with missing observations or those arriving at random intervals. Although initial research on continuous-time models has been conducted using neural ordinary differential equations [82], further exploration is required to expand this approach for datasets featuring complex inputs (e.g., static variables) and to compare them against established models. Additionally, as highlighted in [83], time series often exhibit a hierarchical structure with logical groupings between trajectories, such as in retail forecasting where product sales in the same geography can be influenced by common trends. Consequently, there is potential for enhancing forecasting performance by developing architectures that explicitly consider such hierarchies, representing an intriguing research direction compared to existing univariate or multivariate models.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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