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An Induction on Semi-Regular Group Divisible Design

K. K. Singh Meitei

Abstract: **A Group Divisible design (v, b, r, k, λ_1, λ_2; m, n) is said to be Semi-Regular when r- $\lambda_1 > 0$ and rk – vλ_2 = 0. In this paper it is proposed that starting from Semi-Regular Group Divisible (SRGD) design with k = m, λ_1 = 0, many series of Group Divisible (GD) designs are constructed without disturbing its Semi-Regular property. Such design are useful in the formation of different plats of same size reinforced Cement concrete, by having choice of different iron bars of different gauges by civil engineers.**

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I. INTRODUCTION

Plackett and Burman [1] have given a series of Orthogonal arrays [4λ, 4λ–1, 2, 2] for all integral λ≤ 25, except λ = 23 from which Semi-Regular Group Divisible (SRGD) designs with λ_1= 0 can be constructed. For the existence of Group Divisible (GD) design, Bose and Connor [2] have proposed many relations among the parameters of the design. Using some combinatorial methods and the orthogonal arrays, many series of GD designs are presented in the literature of Bose, Shrikhande and Bhattacharya [3]. Many works are available in the book written by Raghava Rao [5]. Two of many contributions proposed by Kageyama and Tsuji [7] i.e., (i) a GD design is Semi-Regular iff k/m is an integer and every block contains exactly k/m treatment(s) from each group of the association scheme and (ii) a GD design is singular iff k/n is integer and every block contains exactly k/n groups of the association scheme, are enough to mention. Dey and Nigam [8] has suggested a construction method of a GD design (v = v′/s, b = b′/s, r =r′, k =k′, λ_1 = 0, λ_2 = sλ_2'; m =m′, n = t), starting from another GD design (v′ = m′n′, b′, r′, k′, λ_1' = 0, λ_2'; m′, n′ = st); (s ≥ 2, t ≥ 2). If the starting design is SRGD design, the resultant design is also so, by maintaining the same block size of the starting design. Kageyama [9], starting from α-resolvable Balanced Incomplete Block (BIB) design, has constructed α-resolvable regular GD design.

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K.K. Singh Meitei, Department of Statistics, Manipur University, Canchipur, Imphal 795003, India

In the sphere of construction of GD designs, Banerjee and Kageyama [10] have contributed a method of constructing Regular Group Divisible (RGD) design. A series of RGD design (v^*= v-1, $b^* = r(v-k+1), r^* = (r-1)(k-1)+1, k^* = k-1, \lambda_1^*= 1$, $\lambda_2^*=k-2$) have been proposed by Sastry [11], starting from a BIB design (v, b, r, k, $\lambda = 1$). In the literature of Kumar [12], a method of construction of GD designs from a given BIB design, using Kronecker products which were introduced by Vartak [4] is presented where the constructed PBIB designs, even though they are less efficient than the parent BIB design, becomes necessary for different combinations of "r" (replications) and "k" (block size).

II. CONSTRUCTION

In the following starting from a given SRGD design (v=mn, b, r, k=m, λ_1= 0, λ_2; m, n), the construction of another SRGD design (v^*= (m+1)n, $b^* = nb, r^* = nr, k^* = m+1, \lambda_1^*$= 0, λ_2^*= nλ_2; m, n) follows.

A.Theorem: The existence of a SRGD design (v=mn, b, r, k=m, λ_1= 0, λ_2; m, n) implies that of a SRGD design (v^*= (m+1)n, $b^* = nb, r^* = nr, k = m+1, \lambda_1^*$= 0, λ_2^*= nλ_2; m, n).

Proof:- Let $\theta_{n(i-1)+j}$ be the j[th] treatment in the i[th] group of the Group Divisible association scheme on which the given SRGD design (v=mn, b, r, k=m, λ_1= 0, λ_2; m, n) based on; i = 1, 2, … , m; j = 1, 2, … , n. That is, $G_1 = \{\theta_1, \theta_2, \ldots, \theta_n\}$, $G_2 = \{\theta_{n+1}, \theta_{n+2}, \ldots, \theta_{2n}\}$, …, $G_m = \{\theta_{(m-1)n+1}, \theta_{(m-1)n+2}, \ldots, \theta_{mn}\}$. Augment a new group $G_{m+1} = \{\theta_{mn+1}, \theta_{mn+2}, \ldots, \theta_{(m+1)n}\}$ to the m given groups viz.; $G_1, G_2, \ldots, G_m$. Further, a new block is given by

$$B_{lj} = B_l \cup \{\theta_{mn+j}\} \ldots. \quad (1)$$

where B_l is the l[th] block of the given SRGD design, l = 1, 2, …, b; j = 1, 2, …, n.

In addition to the mn treatments of the parent SRGD design, belonged to $G_1, G_2, \ldots, G_m$; the n new elements belonged to G_{m+1} are incorporated in both in the association scheme and in the resultant design. Thus v^*= (m+1)n. As l takes the values 1, 2, …, b and simultaneously j takes 1, 2, …, n, the total number of new blocks B_{lj}'s is equal to nb. That is $b^* = nb$. The block size of the new block is equal to $||B_{lj}||$ = $||B_l U\{\theta_{mn+j}\}||$ = (m+1) as $\theta_{mn+j} \neq \theta_{(i-1)n+j}$; i = 1, 2, …, m, where by ||A||, it denotes the cardinality of a given set A. Since a treatment θ, in the parent SRGD design gets replicated r times, there exist r blocks, say, $B_s(\theta)$, containing θ; s = 1, 2, …, r. By the construction method of new block i.e. $B_{sj} = B_s(\theta)U\{\theta_{mn+j}\}$ defined in (1), all the new blocks B_{sj}'s contain θ. Each of the treatments $\theta_1, \theta_2, \dots, \theta_{mn}$ from the parent design gets replicated rn times in the resultant design as j = 1, 2, …, n. For a treatment $\varphi \in G_{m+1}$ the newly constructed blocks, say, $B_{l\varphi} = B_l U\{\varphi\}$, contains φ in the resultant design. As l = 1, 2, …, b, the number of newly constructed blocks B_{lj} containing φ, is b i.e., rn as v=mn, k=m and vr = bk. Then every newly introduced treatment θ_{mn+j} , gets replicated rn times. Therefore, $r^* = nr$.

By the construction method of new block B_{lj} defined in (1), it is cleared that no two treatments belonged to the same group G_i (i = 1, 2, …, m) occur together in any new block B_{lj} and also no two treatments belonged to the group G_{m+1} have been incorporated in the new block defined in (1) at the same time. So no two treatments belonged to same groups $G_1, G_2, \dots, G_{m+1}$ occur together in the resultant design. Thus λ_1^*= 0.

For counting the number of times of concurrence of two treatments θ, φ belonged to two different groups (i.e., θ, φ do not belonged to same group), we classify all possible cases into two exhaustive cases: (i) θ, φ belong to $G_1, G_2, \dots, G_m$, but not to a same group (ii) one of θ, φ (θ, say) belongs to $G_1, G_2, \dots, G_m$ and another φ belongs to G_{m+1}.Under the case (i), there are λ_2 blocks, say, $B_f(\theta,\varphi)$; f = 1, 2, …, λ_2, containing θ, φ together in the parent design. From each of $B_f(\theta,\varphi)$'s, we can construct $n\lambda_2$ blocks, $B_f(\theta,\varphi)$ U $\{\theta_{mn+j}\}$ as blocks of the resultant design; j = 1, 2, …, n. As j takes 1, 2, …, n, the two treatments θ, φ belonged to two different groups of $G_1, G_2, \dots, G_m$, get replicated together in $n\lambda_2$ blocks of the resultant design. Further, under the case (ii) there are r blocks, say, $B_s(\theta)$ }; s = 1, 2, …, r, containing $\theta \in G_i$ (i = 1, 2, …, m). By the construction method of new blocks of the resultant design defined in (1), each of all the r new blocks $B_s(\theta)U\{\varphi\}$; s = 1, 2, …, r, contain θ and φ together. Thus in the resultant design, θ, one of two treatments belonged to either $G_1, G_2, \dots$, or, G_m and another treatment φ belonged to G_{m+1}, appear together in r (i.e., $n\lambda_2$ as the design is Semi-regular i.e., $v\lambda_2 = rk$) blocks of the resultant design. Thus λ_2^*= $n\lambda_2$. Hence the proof is complete.

A.Remark: An increase of block-size of the parent SRGD design by 1, makes an increase of treatment numbers of the parent SRGD design, by n. It privileges the experimenters to increase the treatment number without much affecting the block-size wherever necessary.

Applying p times the same construction method of new blocks of the resultant SRGD design and the same augmenting method of groups on which of the design bases, a corollary is immediate.

A. Corollary: The existence of a SRGD design (v=mn, b, r, k=m, λ_1= 0, λ_2; m, n) implies that of a SRGD design (v^*= (m+p)n, $b^* = bn^p, r^* = rn^p, k^* = m + p, \lambda_1^*= 0, \lambda_2^*= \lambda_2 n^p$; $m^* = m + p, n^* = n$).

Using a SRGD design (v=6, b=8, r=4, k=3, λ_1= 0, $\lambda_2 = 2$; m=3, n=2), Reference number SR19, Clathworthy [6], the construction of another SRGD design (v^*= 8, $b^* = 16, r^* = 8, k^* = 4, \lambda_1^*= 0, \lambda_2^*= 4, m^* = 4, n^* = 2$), Reference number SR39, Clathworthy *ibid*, is shown as an example of the A. Theorem, at below.

A. Example: The 8 blocks of the given SRGD design (v=6, b=8, r=4, k=3, λ_1= 0, $\lambda_2 = 0$, m=3, n=2) and the 3 groups of the Group Divisble association scheme on which the given SRGD design bases, are B_1= {0, 2, 4}, B_2= {0, 2, 5}, B_3= {0, 3, 4}, B_4= {0, 3, 5}, B_5= {1, 2, 4}, B_6= {1, 2, 5}, B_7= {1, 3, 4}, B_8= {1, 3, 5} and G_1= {0, 1}, G_2= { 2, 3}, G_3= {4, 5}respectively.

By the construction method of new blocks of the resultant SRGD design and the augmenting method of new groups on which the resultant SRGD design bases, we get the 16 blocks and the 4 groups of the SRGD design (v^*= 8, $b^* = 16, r^* = 8, k^* = 4, \lambda_1^*= 0, \lambda_2^*= 4, m^* = 4, n^* = 2$) as given at below.

B_1= {0, 2, 4, 6}, B_2= {0, 2, 5, 6}, B_3= {0, 3, 4, 6}, B_4= {0, 3, 5, 6}, B_5= {1, 2, 4, 6}, B_6= {1, 2, 5, 6}, B_7= {1, 3, 4, 6}, B_8= {1, 3, 5, 6}, B_9= {0, 2, 4, 7}, B_{10}= {0, 2, 5, 7}, B_{11}= {0, 3, 4, 7}, B_{12}= {0, 3, 5, 7}, B_{13}= {1, 2, 4, 7}, B_{14}= {1, 2, 5, 7}, B_{15}= {1, 3, 4, 7}, B_{16}= {1, 3, 5, 7} and G_1= {0, 1}, G_2= { 2, 3}, G_3= {4, 5}, G_3={6,7} respectively.

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K. K. Singh Meitei currently is a Associate professor, Department of Statistics, Manipur University, Pin-795003,India.. He is M.A. &, Ph. D. He has published 17 research papers and also he is the author of two books.

